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CONCEPTUALIZING MEANING

Abstract. Descriptions have been the object of attention of many philosophers. The goal of this article is to inquire into the meaning of those descriptions which, due to the peculiar character of the objects of description, have been interpreted in different ways, and to investigate in which sense one is able to speak of the existence (or non-existence) of an object of description. The various sorts of descriptions are inquired into; the question which entities exist and which do not is dealt with, and, in relation to this, how 'meaning' is to be understood.

Introduction

The question of what meaning is has led to a large number of approaches. Where may meaning be found? How can it exist? In this article, these questions will be dealt with. In any enquiry it is of importance to try to find a theory that does not rest upon suppositions which cannot be justified, or at least clarified.

When meaning is the subject of enquiry, this entails that theories which defend the existence of a meaning existing independently of (users of) language are to be investigated. This is done in the first section. It has a wider scope, however, dealing with the various sorts of objects which may involve meaning. As it will turn out, it is not enough to discuss a reference and a meaning; a large number of situations cannot be explained without a third element, a secondary reference, as I shall call it.

Section 2 deals with some particular problems concerning descriptions of a difficult nature. In order to demonstrate these problems, a logical analysis is presented in section 3. An attempt to solve a number of problems which arise from the ambiguity that is characteristic for natural language is undertaken by this means as well. I have, throughout this article, tried to take a critical and cautious stance; I prefer a lack of a large number of certainties to a situation in which the price for the desired results is a dogmatic attitude.

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1. The place of meaning

1.1.

When a description of something is given, an important question connected with this act is whether the thing or person described exists. Few problems present themselves when one is dealing with common descriptions like ‘this man’, ‘the president of the U.S. in 1863’ and ‘the author of *Moby-Dick*’. However, it is possible to distinguish statements that do not render contradictions when compared with external reality but do not refer to an external object.

Bolzano claims that there are certain conceptions to which no object corresponds.[1] He gives as examples Nothingness, a green virtue, a round quadrangle and a golden mountain. Russell’s claim, that these ‘objects’ infringe upon the law of non-contradiction,[2] seems to me to be too narrow a point of view: it is the case for some things, but not for all. I think the so-called things mentioned should not be considered all at one and the same level; there are important distinctions to be made. Keeping these in mind, I shall distinguish three levels here; it is possible that there exists a greater number than this, but I won’t deal with that here; in fact, only the third and, to a lesser extent, the second level are of importance.

The first level at which non-existing things can be classified deals with so-called things that cannot be expressed particularly, such as Nothingness. This sort of so-called things is irrelevant for this article and will receive no further attention. The second level is that of the impossible so-called things. A distinction must be made here between things which are impossible because of their nature on the one hand and those which are so because of their essence. Obviously, a green virtue (an example at the first level) cannot exist: a virtue, an abstract value, cannot have a colour, after all;[3] by attributing ‘green’ to a virtue, one makes a category mistake. ‘A round quadrangle’ (an example at the second level) is impossible for another reason. Here, an object which is essentially formed with four angles is concerned. If it ceases to have four angles, it ceases to be what it is, namely a quadr-angle. A round quadrangle, one may say, contradicts external reality.[4]

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[1] Bolzano (1985): pp. 112, 113 (§ 67).

[2] Russell (1905): pp. 482, 483.

[3] I will forgo a discussion whether the word ‘virtue’ has a meaning at all here.

[4] I readily grant that this is crudely formulated: ‘contradicting external reality’ is a (very) vague phrase. I have nonetheless used it here since the focus in this article is on the philosophy of language and not on epistemology; formulating a precise position would merely needlessly complicate things.

The third level is concerned with non-existing things that do not contradict external reality. It may not be possible to find a golden mountain in the outside world, but it is not inconceivable that one exists (one may conceive a possible world in which at least one golden mountain exists).

This also applies to 'the present king of France'. Only when one knows about the form of government of France is one able to determine that the description does not correspond to external reality. Clearly, propositions concerning so-called things at the third level are not as easily dismissed as those at the second are. It is useful to examine Meinong's attitude concerning this matter. He presents a theory according to which the things whose existence is denied somehow 'exist' nonetheless: "[...] If I should, regarding an object, be able to judge that it does not exist, I apparently first somehow have to grasp the object, in order to state the not-being of it, more precisely to predicate it to it, or deny it of it."[5]

According to Meinong, there must be 'Aussersein' (literally: 'outside of being'),[6] a situation in which the thing neither exists nor does not exist – the existence of a thing is external to it[7] – and which forms the vestibule, as it were, of judging, on the basis of which it becomes apparent whether the thing exists or does not exist (outside of Aussersein).

1.2.

Having established the various levels, it is now possible to inquire critically into the instances where meaning is possible. Concerning the things at the second level, Meinong's argumentation cannot be accepted: it is not the case that one has a notion of a round quadrangle of which one subsequently denies that this represents a quadrangle existing in the outside world. What Bolzano, as a logical realist, has to say about this, that to these things corresponds a 'Vorstellung *an sich*',[8] (a representation as such) is not tenable, in my opinion: a notion always needs a subject imagining it. Be that as it may, a more serious problem presents itself: one has to imagine a round quadrangle.

According to Bolzano, saying 'a round quadrangle', one does not claim anything, but a representation is created (in the mind) (Bolzano uses the word 'vorgestellt').[9] This is impossible, however. As soon as one tries to

5 Meinong (1971): p. 491.
6 Meinong (1971): ibid.
7 Meinong (1971): p. 494.
8 Bolzano (1985): p. 112 (§ 67).
9 Bolzano (1985): p. 103 (§ 19).

imagine the quadrangle as round, it ceases to be a quadrangle. Now the question emerges how one is capable of forming the sentence: ‘There are no round quadrangles.’ After all, if one is not capable of forming a notion of round quadrangles, the sentence seems to have no meaning. One does not even know what one is thinking (as there is nothing to think about in this case).

Indeed, I do not think the sentence has meaning. I will return to this point later. At the moment, it’s useful to analyse the sentence. ‘There are no round quadrangles’ consists of four parts: ‘there’, ‘are’, ‘no’ and ‘round quadrangles’. What is imagined when one is confronted with the first three parts? Nothing, really: they are words which serve a function within the sentence. ‘No’ is merely a negative element and ‘there’ indicates ‘anywhere’ (which does not excite a representation since anyone’s scope is limited and does not comprise the entire universe). ‘Are’ is a difficult part of the sentence, but obviously no representation corresponds to this. These parts do not necessarily have to have a meaning, just as long as the sentence has one. So the sentence ‘there are no paintings in this room’, e.g., may have a meaning. In ‘there are no round quadrangles’, however, the crucial element is problematic.[10]

This argument would perhaps suffice, were it not that ‘round quadrangles’, other than the other parts of the sentence, is the composition of an adjective and a noun and a thing corresponding to it is expected here. In section 1, I indicated why ‘a round quadrangle’ cannot exist. The description does not excite a representation, either. In fact, ‘a round quadrangle’ is only a composition of two words that has no function. It can be called a ‘flatus vocis’: the words exist, but nothing corresponds to them. The fact that the first three words do not invoke a representation is not problematic, as long as the thing described does: the sentence ‘there are no unicorns’ has meaning (provided one imagines something corresponding to ‘unicorns’).

Things at the third level cannot be dealt with this easily. It is important to distinguish two sublevels within the third level. An example of the first is the beforementioned ‘golden mountain’. Assuming that such a mountain cannot be found anywhere, the question arises what the status of the statement ‘There is a golden mountain’ is. One can imagine such a mountain.

[10] For the sake of clarity, I contrast this example with another one. The sentence ‘there are no chiliagons’ does have a meaning. In this case, no representation exists (I, at least, fail to excite one), yet the sentence can be used by substituting a polygon one *is* able to imagine for the chiliagon. Similarly, Descartes distinguishes between imagining and understanding a chiliagon, being able to do the latter while being unable to do the former (R. Descartes, *Meditationes de Prima Philosophia*, pp. 72, 73 (Meditatio 6)).

Does this mean that it exists 'ausser sein'? By no means. When one imagines a golden mountain, a mountain is imagined, examples of which have been seen, or descriptions of which have at least been heard or read, whereupon the predicate 'golden' is connected to this representation. 'Golden mountain' is a construction from these two notions, just as (I assume) 'unicorn' is a construction based on the representations of a horse and a horn.

Meinong makes an epistemological shift, supposing that the composition '(a) golden mountain' would exist primarily. Instead of thinking that a notion of a golden mountain is created on the basis of the experience of one or several mountains on the one hand and the colour golden on the other, Meinong starts on the other side and presupposes the golden mountain as a whole.

This objection cannot be maintained concerning the second sublevel. 'The king of France in 2010',[11] abbreviated hereafter as 'Kf 2010', is not the result of abstraction from two or more concepts. In order to establish whether one imagines something when saying 'Kf 2010 is bald', an enquiry into the meaning is required. According to Russell, this proposition does not refer, but it does have a meaning. I will argue that one can only speak of meaning when there is a reference; however, 'reference' must be understood in a broader way than is usually done.

When the sentences 'The president of the U.S. in 2010 is bald' and 'Kf 2010 is bald' are verified, the first sentence turns out to be false. What about the second one? According to Russell's approach, it is false as there was no king of France in 2010.[12] Strawson refutes this thought: "[...] The question of whether [someone's] statement [that the king of France is bald, or, as in Strawson's example, wise (which one of these properties is attributed is not important for the example)] was true or false simply *didn't arise*, because there was no such person as the king of France."[13]

What Strawson says here is important, but not radical enough. He maintains Russell's statement[14] that the statement has a meaning.[15] What does this comprise, however? Suppose an unmarried man is invited to a gathering and is mistakenly asked: 'Will your wife be there as well?' According to Russell's and Strawson's analyses, this sentence has a meaning, despite the

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[11] I prefer to use this description to the one used in section 1 ('The present king of France'), since the latter is bound by context.

[12] (nor in, for example, 1950, when Russell was still alive).

[13] Strawson (1950): p. 330.

[14] Russell (1993): p. 179; Russell (1905): pp. 483, 484.

[15] Strawson (1950): p. 331.

fact that it does not refer. However, the bachelor, on hearing 'your wife', has no representation of his wife (as there is none) and neither does the inquirer: he simply states a question, the form of which is the same as it would be if it had been asked to someone else whose marital status is unknown to him. The question would, by contrast, have had a meaning if it had been directed at a married man, in such a way that the inquirer not only knew that he was married, but also knew his wife or had at least heard or read a description of her (and had a representation of her when he asked the question). Here, however, the words are, one could say, empty. Nothing is represented and the sentence accordingly has no meaning.

In order to illustrate his statement that 'Kf 2010' has a meaning, Strawson gives an interesting example:[16] One may tell a story about the king of France and ascribe all sorts of predicates to him. In this case, one has a representation (of the king of France). It is important to distinguish the representation of the speaker from the one the hearer has. It is either the case that the speaker has a certain representation and gives information of this to the hearer, who, on this basis, creates a representation of his own until this is sufficient, or that the speaker states something about a thing which has no existence in external reality, the situation being one in which the speaker has no representation, and the hearer creates a representation which ceases to be at the moment information which is not conformable to external reality is provided.

An example of the first situation is the representation the speaker has of Lincoln. He has a lot of information about this person at his disposal and has a representation of Lincoln on the basis of this. If he presents this information to the hearer and the latter understands it, a representation will arise with him as well (which will, however, probably not be the same as the one the speaker has). On the basis of statements like 'He was the 16th president of the U.S.', 'He was president during the Civil War' and 'He was assassinated in 1865', one forms a representation of the historical person. Even if a great number of historical data would turn out not to be correct, the representation would remain.

Strawson's king is an example of the second situation. If only a mythical, legendary or otherwise fictive king (fictive in the sense that he has no existence in history) is concerned, no problem concerning the forming of a representation need arise: one can imagine a man living in a palace, being wise, being bald or not and having other properties than these. However, if

[16] Strawson (1950): ibid.

this king is supposed to be the king of France in 2010, something peculiar happens.

The speaker may inform the hearer about such a king and produce a representation of a king. If he then states: 'This king of whom I spoke is, by the way, the king of France in 2010', this representation cannot be maintained by the hearer. If he knows about France's form of government, he will separate the representation from one about the king of France, so the representation is not about the king of France. If he does not know about it, the representation won't be affected, but it won't be adequate as it is not about the king of France. Consequently, 'Kf 2010 is bald' does not, in contrast to what Strawson and Russell state, have a meaning.

It may be useful to maintain another way of referring than the one which is usually utilised. Frege distinguishes between meaning ('Sinn') and reference ('Bedeutung'). This distinction is valuable, but, in my opinion, not sufficient. If one wants to determine the meaning of a sentence, one is dependent on a reference of some parts of the sentence. I will henceforth call this 'secondary reference'; 'reference' will mean the same as it does with Frege.

In Frege's example, 'Odysseus was set ashore at Ithaca while sound asleep,'[17] although 'Odysseus' has no reference, the sentence has a meaning. So, in order for a sentence to have a meaning, a reference is not necessary. Still, upon hearing the sentence, one must have a representation of Odysseus; this representation is the secondary reference.

This is the difference between 'Odysseus' and 'Kf 2010'. In the first case, something is represented; if this is not the case, the sentence has no meaning. When someone hears about Odysseus for the first time, the sentence in which the word 'Odysseus' occurs may receive meaning: 'Odysseus sees that the Cyclops is coming' gives him enough information to conclude that Odysseus is some being. One may create a representation and conclude that Odysseus is a man, woman, or even an animal, but at least one has a representation. The sentence 'Odysseus was seen by Hector' leaves open a greater number of options. Odysseus may as well be a person or animal as an inanimate object. If one connects a representation to the name, the sentence does receive meaning.

In the case of 'Kf 2010', this option is not present. Here, the notion itself is problematic. 'Kf 2010' cannot simply have as its secondary reference a representation of a man, woman, animal or object, since the representation

[17] Frege (1892): p. 32.

is one that can never have an equivalent in external reality: if it is properly understood, one sees and acknowledges that it cannot be maintained. (At least in reality as I know it. In a possible world in which France was a monarchy in 2010, 'Kf 2010 is bald' may of course have a meaning.)

Again, 'Lincoln' has a reference, 'Odysseus' has a secondary reference (whether he has a reference or not, one has a representation of him) and 'Kf 2010' has neither. 'Lincoln' has, besides a reference, a secondary reference for those who know him, having arranged the information into a representation. Only sentences containing something which may have a secondary reference have meaning. This meaning is attributed to them by someone hearing or reading them. Meaning cannot, at least not by me, be said to be something that exists irrespective of human efforts to understand and deal with language. Meaning does not play a role when someone tries to determine what a sentence means; it may be possible that meanings in the sense of independently existing entities exist, but whether they do or not seems to be irrelevant. The basis for meaning lies in the existence of secondary references, which makes it possible for the same sentence to have meaning for one person and fail to have one for another.

There are, however, situations in which no secondary reference and hence no meaning can be rendered. 'Kf 2010' is an example. This phenomenon is also found in the structure of natural languages. In many languages a so-called overt subject is used in simple sentences like 'it rains' (or: 'it is raining') ('es regnet', 'il pleut'). Italian, however, lacks such a subject. To express that it is raining, the statement 'piove' suffices (the ending makes clear that the third person singular is concerned). This means that the overt subject ('it', 'es', 'il') really has no meaning. Speakers of English, German or French have no notion of some thing that rains. It consequently has no secondary reference and therefore no meaning. The only secondary reference here lies in the 'raining' itself: here, something is imagined. 'It rains' is, as it were, a petrified expression: one simply uses it, without wondering what the meaning of 'it' is. In this situation, of course, no problems arise: such expressions merely serve as a means to convey information and there are fewer demands imposed on natural languages than on formal ones.

Sentences only have a meaning when the thing described has a secondary reference. Of course, a sentence can be well formed without having a meaning, so this is not a sufficient condition. 'Kf 2010 met the president of the United States of 2010' is well formed, but has no meaning. Hence, the distinction between language and external reality has to be acknowledged. When language describes external reality adequately, a representation of things in external reality and, hence, a meaning is produced.[18] So, a secondary reference, be it based on a reference (e.g. ‘Lincoln’) or not (e.g. ‘Odysseus’) is a prerequisite for meaning.

2. Implications and problems

2.1.

In section 1, secondary reference, which deals with representations, has received attention. It has to do with the manner in which an individual meaning is produced: for the speaker, the sentence he utters has a meaning if it has a secondary reference.[19] It is now to be examined what happens when communication takes place. When two people talk to each other about something which gives them both a secondary reference, will there be a single meaning (for both)?

Russell rightly points out that descriptions vary for different people.[20] His elaboration for descriptions of historical people is consistent and may be maintained. However, this elaboration holds only for cases in which objects with a reference are dealt with. According to Russell, no object corresponds to a word like ‘Odysseus’. Since he indicates that the only thing constant in different circumstances is the object, his theory becomes problematic for all sentences with a meaning and without a reference. I have tried to solve this problem by introducing a secondary reference.

When people communicate, there has to be some element which is constant (something they can share), otherwise communication would be impossible. What is this element? In my opinion, it is that which is communicated. When two people are talking about ‘Odysseus’, they may have different representations while talking and nevertheless be able to have a conversation about him. As long as the things discussed do not concern the representations, no problem in communication need arise. Person A may have a representation of ‘Odysseus’ according to which he is malevolent, person B may

[18] I have not explored the difficulties which accompany this position or considered its alternative, that (external) reality is (partly) determined by language, since it would deviate too far from the matters discussed here. That does not mean that it is not an important issue, but it may very well be undeterminable.

[19] This is a *necessary* condition, not a *sufficient* one. To illustrate this by means of the example I mentioned at the end of the previous section, the sentence ‘Kf 2010 met the president of the United States of 2010’ contains a secondary reference (‘the president of the United States of 2010’ has a secondary reference) but lacks a meaning (because the other crucial part of the sentence, ‘Kf 2010’, lacks a secondary reference).

[20] Russell (1982): pp. 29, 30.

have one according to which he is not. Only when some property is dealt with, for example his cunningness, need they agree.[21] They can both cling to their own representations about other properties, such as his alleged malevolence.

By connecting meaning to a representation, the objection that experts decide on issues with which other people are unfamiliar, such as distinguishing certain species of trees,[22] is resolved. This way of looking at things puts fewer constraints on the contents of communication than most theories do. I think it is plausible to deal with communication like this: in what may be called an occamian approach, one may state that all that is required for communication is present; there is no need to posit any further assumptions. Only if higher demands than necessary are made regarding the content need problems arise.[23]

The alternative view that use is (in many cases) the crucial element[24] may be said to be unproblematic for some situations, namely those in which one does not reflect on one's words before actually communicating them (and the utterance is a spontaneous one) and in cases where the meaning is not an issue (in cases where phrases are used metaphorically, for instance). Similarly, a computer (presuming that it does not reflect) may convey a message that is understood by the person reading it without the computer itself understanding what it has communicated. It does not, then, have a meaning for the computer.[25]

2.2.

I will now devote some attention to problems which may arise when things are understood differently by different persons (e.g. when person A has a secondary reference and person B does not). Ambiguity may play a role in sentences of a natural language. 'Smith's murderer is insane' can be interpreted in two ways: either the person talked about is Smith's murderer and happens to be insane as well, or Smith's murderer, who-ever he is, is insane (as he murdered Smith). This distinction was made by Donnellan, as the referential and the attributive use of definite descriptions, respectively,[26]

[21] They don't have to agree with regard to the issue whether he is cunning or not, of course; the point is that the conditions to be able to bring up this issue in the first place have to be present.

[22] Putnam (1995): pp. 18, 19.

[23] Cf., e.g., Putnam (1989): p. 25.

[24] Wittgenstein (1997)(2): p. 262 (§ 43).

[25] Cf. Searle (1984), pp. 31–34.

[26] Donnellan (1966): p. 285. The distinction can, incidentally, be traced back to the Scholastic classification of 'de re' and 'de dicto'.

but in my opinion there is no real problem here. The ambiguity is caused by the fact that natural language admits such differences. When the sentence is analysed with the aid of logic, as will be done in the third section, this ambiguity is solved.

There is, however, another problem. This is about entities which lack a simple status in existence. 'God' is an example. Does 'God is not malevolent' have a meaning or not? A number of interpretations are possible. One comprises that the speaker thinks that God does not exist (and therefore is not malevolent). In this case, the sentence has no meaning: there is no entity which is supposed to be malevolent, nor is the representation of 'God' present for the speaker. 'God' is simply a word and there is no meaning involved since a secondary reference is absent. Another interpretation is the following: God exists and is not malevolent. Now, because of the first part of this conjunction ('God exists'), the sentence has a meaning if the speaker believes that God exists and lacks one if he does not.

The description receives a status which depends on the opinion of the person describing: if he believes in God, the description has a meaning and is 'true' respectively 'false', depending on the belief of the person describing, according to whom He is malevolent or not.[27] If he does not believe in Him, it has no meaning and is neither 'true' nor 'false'.

The sentence has a meaning if one thinks that God exists and (therefore) has a representation of God (be it limited). Can God be described, however? Of course, some properties of God can be named, but this does not suffice. He has a particular nature and simply naming some properties is not enough to provoke a representation. In the case of 'Odysseus', it is enough to state that he is a cunning man, who has travelled widely, met a number of strange creatures and did battle with many men. On the basis of these facts, a representation can be produced. Where information is lacking, an abstraction from actual persons one knows may fill in the gaps (at least in order to create a representation).

In the case of God, this option is not present: there are no beings like Him (at least none with whom one has, I assume, any acquaintance), so all of His properties have to be known in order to create an adequate representation. A description of God can, accordingly, not be given. Sentences containing 'God' have a particular status: one cannot simply state that they have a meaning (since a complete description and with it a complete

[27] A problem here is that this way of thinking leads to a situation in which the person describing determines reality; in order to avoid this, I have not treated 'truth' as an absolute concept.

representation is missing), nor that they lack one (since some people form a representation of God, even if this is limited and does not do justice to the object represented in the case it exists). In the last section, this will be analysed with the aid of logic.

3. The limits of meaning

An elementary analysis, which will now be presented, is created for four sorts of descriptions: those which have a meaning, those which lack one, those which may receive a meaning depending on the point of view of the hearer and those of which it cannot be determined whether they have a meaning or not.

Ambiguities in natural languages are solved when this analysis is applied. 'Smith's murderer is insane', from the previous section, is ambiguous until the sentence is properly analysed. In the elementary analysis, one has to opt for either 'the person who killed Smith is insane' or 'that person, who (by the way) killed Smith, is insane.'

In the analysis, the following result is found:

1. The person who killed Smith is insane:

   $$(\exists x)((Mx) \wedge (Mx \rightarrow Rx) \wedge (\forall y)(My \rightarrow y = x)),$$

   where 'M' stands for 'murderer of Smith's' and 'R' for insane';

2. That person, who killed Smith, is insane:

   $$(\exists x)(((Mx) \wedge (\forall y)(My \rightarrow y = x) \wedge Rx))$$

Real problems do not emerge until the presence or absence of meaning is doubtful. When dealing with a secondary reference one may, after having applied the elementary analysis, add a symbol to the description in order to state that it has a meaning. I will use the Greek letter μ for this. Of course, this is not a symbol in predicate logic. Logic does not deal with meaning; it merely gives adequate descriptions. (This point was already made by Wittgenstein: logic does not say anything;[28] it is simply a condition to be able to say something.)

The elementary analysis of 'the 16^{th} president of the U.S. had a beard' will be:

$$(\exists x)((Px \wedge (\forall y)(Py \rightarrow y = x) \wedge Bx) \wedge \mu),$$

[28] Wittgenstein (1997)(1): p. 54 (§ 5.43).

where 'P' stands for 'the 16$^{\text{th}}$ president of the U.S.' and 'B' for 'has (or had) a beard'.[29]

The absence of meaning is represented by '$\neg\mu$'. So, 'the king of France in 2010 is bald' is represented thus:

$$(\exists x)((Kx \wedge (\forall y)(Ky \rightarrow y = x) \wedge Qx) \wedge \neg\mu),$$

where 'K' stands for 'the king of France in 2010' and 'Q' for 'bald'. It may appear that 'there is a king of France in 2010, and he is bald' is incompatible with the fact that this description has no meaning. However, 'there is' is not to be understood ontologically, but semantically: it does not imply an existence, but merely conveys that the sentence is at this time a candidate for receiving a meaning. 'There is' does not mean anything by itself here, but may be used meaningfully in some contexts (e.g. by saying: 'there is a chair in this room' when there is one (cf. the example of 'there are no paintings in this room' from section 1.2); there are, in fact, many words which merely serve as a means and have no meaning themselves). If, however, something which lacks a secondary reference appears after 'there is', this possibility vanishes.

Meaning cannot be acknowledged or denied as easily as in the situations displayed above in every case. The sentence 'God is not malevolent', from the previous section, is an example. I have already indicated where the difficulty lies. To elaborate on this point, at least three interpretations of the sentence can be discerned.

The first one is fairly simple: it is not the case that God exists; therefore, he is not malevolent. This sentence has no meaning, since one has no representation when saying or hearing 'God' (no secondary reference is involved):

$$\neg(\exists x)((Gx \wedge Sx) \wedge \neg\mu),$$

where 'G' stands for 'God' and 'S' for malevolent. This interpretation is not very likely to occur.

The second interpretation is more complex: God exists and is not malevolent. The sentence does not simply have a meaning, based on a secondary reference: whether it has one or not depends on the conviction of the person dealing with it. Here, a conditional meaning is the case: conditional because the speaker's or hearer's point of view determines the presence or absence of meaning and no absolute statement is the case here:

$$(\exists x)((Gx \wedge \neg Sx) \wedge (\mu \vee \neg\mu)).$$

29 Lincoln did not have a beard in his younger years, but that does not matter for the example.

The last and most difficult interpretation is the following: It is not the case that there is a God who is malevolent. This interpretation cannot, as will become clear, be properly understood. In order to form the sentence, no representation has to be created. This requirement is present, however, in order to determine whether the sentence has a meaning. It cannot be done here, since ‘God’ is, as it were, conditional: only a negative statement is made, in which ‘God’ is enclosed. So, determining whether the sentence has a meaning is impossible:

$$\neg((\exists x)(Gx \wedge (\forall y)(Gy \rightarrow y = x) \wedge Sx)(\wedge\mu?)).$$

In this sentence, a meaning can neither be acknowledged nor denied. The sentence cannot even be understood, as an analysis is required which one does not seem to be able to perform. This analysis is not simply one of all malevolent things, where it is investigated whether God is one of them or not. The problem here is not that one is not capable of investigating all malevolent things; it is rather that being God involves the absence of malevolence.

‘God is not malevolent’ does not have the same status as ‘I see a round quadrangle’. ‘I see a round quadrangle’ is a demonstrably absurd sentence whereas ‘God is not malevolent’ is not per se; furthermore, ‘God is not malevolent’ is within the scope of the negation ‘¬’, which means that nothing is said – nothing positive, at least: because of this, it is impossible to determine whether the sentence has a meaning. In the second interpretation, although the sentence was within the same scope, this problem did not occur because the sentence was understood differently: in that case, the individual person determined whether the sentence had a meaning.

It turns out that sentences cannot be dealt with by an elementary analysis if it is unclear whether they have a meaning or not. The analysis can solve many ambiguities in natural languages, but its limitations must be acknowledged: its domain is limited to sentences which have or lack a meaning.

Conclusion

In this article, I have tried to show where meaning is to be found and to indicate some of the limitations of describing it. The first point, that of meaningfulness or the lack of it with descriptions in singular cases, which was treated in section 1, indicates that a description at a singular level does not necessarily render a meaning. This is explained more easily when one assumes that meanings are formed in individual situations than when meanings are assumed to exist as separate entities.

In section 2, communicative situations turned out to render problems in many situations as some descriptions are ambiguous and some depend on the convictions the describing parties may have – convictions they do not actually hold (one of the parties describing may believe in God, for example, while another may not).

In the third section, an analysis was presented on the basis of which it was shown that some problems in natural languages may easily be solved while others may not; certain ambiguities disappear when one applies this analysis, but statements which do not simply have or lack a meaning are still problematic. This is caused by the fact that logic has its limitations, since it is not concerned with the content of sentences but merely with the form, whereas establishing whether a sentence has a meaning or not involves an investigation of its content.

The analysis given in this article leaves a number of details to be worked out and the number of questions it raises may surpass that of the answers it provides. It is, furthermore, of course limited to my personal stance; another one, from another point of view, can be maintained as well. That does not mean, however, that anything else has the same value; one has to scrutinise one's claims in order to be able to say whether they are tenable, which some theories do not seem to allow.

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LOGICAL FALLACIES AS CODIFIED WITHIN THE CONCEPTUAL SYSTEM OF THE LVOV-WARSAW SCHOOL

Abstract. The paper attempts to characterize the notion of logical fallacy present in Polish analytical philosophy, especially within the conceptual framework of the Lvov-Warsaw School. This framework is based on general methodological rules of carrying out knowledge-gaining procedures. Three sample ideas significant for this purpose are: Czeżowski's concept of analytical description – as it may be employed in identifying some fallacies of describing and defining, Łuszczewska-Romahnowa's pragmatic account of entailment – as it constitutes a framework for analysing some fallacies of reasoning, and Kamiński's systematization of logical fallacies – as it may be treated as a point of departure for a research project aiming at giving a broad systematization of logical fallacies. Because of some key similarities with current approaches to logical fallacies, the conceptual system of the LWS may play a unifying role in bridging the gap between the study of the fallacies in argumentation theory and in Polish analytical philosophy.

Keywords: logical fallacy, logical culture, analytic description, pragmatic concept of entailment, systematization of logical fallacies.

1. Key tendencies in the contemporary study of fallacies

Amongst a number of definitions of fallacy proposed in the body of literature in contemporary argumentation theory, the term 'fallacy' is generally understood in two ways:

- a formal fallacy in reasoning (Johnson 1987, 241), as illustrated by the following idea:

  We commit a fallacy when we reason or draw conclusions incorrectly (Kahane 1969, p. 244).

- any violation of the rules governing cognitive activities, which appear within the argumentative discourse:

  The term 'fallacy' is our most general term for criticizing any general procedure (or what have you) used for the fixation of beliefs that has an unacceptably high tendency to generate false or unfounded beliefs relative to that procedure for fixing beliefs (Fogelin & Duggan 1987, p. 257).

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Both examples point to the fact that the concept of fallacy is related to the concept of rules (norms, criteria, standards) – not only and not necessarily rules of logic, but also to the rules of rational or reasonable discussion. Hence, it may be said that the general aim of a study of fallacies is providing criteria or standards for good argument (Johnson 1987, p. 246) or to elaborate the goal for a given argumentative procedure, and then to elaborate standards to judge when the goal is attained (Woods 2003, p. 5). Thus, any broad understanding of what (logical) fallacy is should allow us in the most general way to identify fallacies by indicating connections to certain rules (of logic, of discourse, of discussion).

The study of fallacies is considered by some logicians and argumentation theorists to be connected to teaching rather than systematic research (see e.g. Finocchiaro 1981, p. 13; Hołówka 1998, p. 9). One of the reasons for this attitude was given by Hamblin (1970), who criticized the so-called Standard Treatment of the Fallacies present in logic textbooks. The main argument for the devastating portrayal of the Standard Treatment pointed to (a) the lack of precise criteria for distinguishing main kinds of fallacies and (b) the incapability of identifying many types of common logical fallacies.

Hamblin's work motivated researchers to elaborate proper criteria for identifying and classifying fallacies in the early 1970's. The dominant tendency observed in early works in argumentation theory and informal logic was the *fallacy approach* to argumentation. It focused on categorizing argumentative fallacies by means of a list of traditional fallacies such as *argumentum an hominem*, *argumentum ad ignorantiam*, *argumentum ad misericordiam*, the fallacy of equivocation, and so on (Groarke 2009).

Although the *fallacy approach* is still popular (Groarke 2011), it has serious disadvantages. The main difficulty with this approach lies in the fact that it focuses on the negative aspect of argumentation – it spots typical errors which are difficult to define and classify, without any positive reference to norms or criteria useful for identifying fallacies. Hence, recent studies in argumentation theory take an indirect approach to the fallacies – their priority is to propose a repertoire of rules or schemes determining the use of arguments, and later to employ them in identifying typical fallacies.

During the subsequent decades, the fallacy approach to argumentation has been replaced by criterial accounts of argument aimed at giving precise tools to represent argument structures. For example, the pragma-dialectical theory of argumentation (van Eemeren & Grootendorst 1987; 1992; 2004) takes as its starting point the ideal model of a critical discussion (van Eemeren & Grootendorst 2004, p. 96) in order to deal with fallacies as violations of pragma-dialectical discussion rules. Another example is argument scheme theory (e.g., Walton, Reed and Macagno 2008), which focuses on determining the most typical patterns of reasoning and putting forward critical questions which are instructive in fallacy identification. Moreover, some non-deductive patterns of reasoning are specified for inference, conflict, and preference (see e.g. Bex and Reed 2011). Once such patterns are discerned, the identification of typical fallacies (understood as improper patterns) becomes more precise.

A formal-logical approach to fallacies is present in Dale Jacquette's analyses of fallacies in terms of logical invalidities of reasoning (Jacquette 2007; 2009).[1] Sometimes fallacies are analysed in terms of informational shortcuts (Floridi 2009) which are treated as common ways of extracting necessary information in an effective way. This idea is present in some texts which regard deductively invalid inferences (such as denying the antecedent) as inductively proper ones (Stone 2012). The general characteristic of those example approaches to argumentation is the adoption of a set of norms as a point of departure for analysing fallacies.

The given characteristics of the main research strands in the study of fallacies enables the observation of some crucial resemblances between argumentation theory and the study of fallacies in Polish analitycal philosophy, especially the Lvov-Warsaw School (LWS). The LWS was a philosophical movement (1895-1939) concentrated in two main research centres: Warsaw and Lwów (Lvov) (see e.g. Woleński 1989; Jadacki 2009). The flourishing of the school is also labelled *the Golden Age of Science and Letters* (Simons 2002). The main thesis concerning the tradition of the LWS holds that within the logico-methodological studies of the LWS logical fallacies are considered as violations of logical norms, broadly conceived as the rules of formal logic, semiotics and a methodology of science. The discussion of these issues will provide reasons for claiming that in order to build a fallacy theory we should establish sets of rules for particular knowledge-gaining procedures. The following section discusses the point of departure for this discussion, i.e., the concept of logical culture.

[1] I do not claim, however, that this formal approach is sufficient for the systematic study of fallacies. It may be treated as a necessary, but not sufficient tool employed in argument analysis and representation.

2. Logical culture as a framework for analysing fallacies

When discussing the conceptual system of the LWS, Jadacki (2009, pp. 68-76) lists crucial research achievements and unique approaches to language and reasoning in the LWS such as: (a) the tendency of treating language as a tool of cognition, (b) the notion of conceptual apparatus, (c) the idea of applying the rule of conceptual optimization and the rule of precision as crucial methodological rules of inquiry, (d) the concept of metaphor, (e) the distinction between ideal and real notions, (f) the concept of object, (g) the semantic nest of object, and (h) the historical order of concepts. In the present section I will focus on the general conceptual framework which constitutes the point of departure for building the concept of logical fallacy within the logico-methodological tradition of the LWS.[2]

"Logical culture" is one of the most basic terms used in Polish analytical philosophy to refer to the knowledge and skills of logic (see e.g. Wybraniec-Skardowska 2009). The concept of logical culture has been discussed in detail e.g. by Ajdukiewicz (1974) and Czeżowski (2000).[3] For the purpose of this paper I shall briefly present key features of logical culture.

Logical culture is built on two elements: knowledge of logic and the skills of applying this knowledge:

> Logical culture, just as social, artistic, literary or other culture, is a characteristic of someone who possesses logical knowledge and competence in logical thinking and expressing one's thoughts (Czeżowski 2000, p. 68).

This idea of logical culture points to the value of logical thinking, which is one of the most important values in human individual and social behaviour. For example, Ajdukiewicz's understanding of logical culture is clearly expressed in his idea of logical thinking. The concept of logical thinking also shows what is his most general understanding of logic. According to Ajdukiewicz (1957, p. 3) logical thinking is a skill which is possessed by someone, who (1) thinks clearly and consequently, (2) expresses

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[2] Since the tradition of logical studies in LWS associates the concept of fallacy with violations of the norms of logic, hence the common use of the term "logical fallacy". My use of this term is also partly suggested by Whately's distinction between logical fallacies and non-logical fallacies (see, e.g., Hamblin 1970; 169-171; van Eemeren 2001, 144-145).

[3] The concept of logical culture found in the works of Ajdukiewicz is related to his programme of pragmatic logic. For a discussion of Ajdukiewiczian idea of logical culture in the context of pragmatic logic see (Koszowy 2010). For a specification of Ajdukiewiczian idea of logical culture see (Łyczek 2010).

her/his thoughts precisely and systematically, (3) makes proper inferences and justifies her/his claims.

Logical culture is particularly important for human scientific and argumentative activities, because it involves the skill of performing various cognitive and linguistic procedures which play a crucial role in scientific inquiry. Hence, the main point made by Czeżowski is that logical culture is a skill that allows us to think logically, and this skill is built on the knowledge of logic. Czeżowski points to some criteria which tell us what thinking is non-fallacious by further explaining his initial definition by giving a list of procedures which constitute logical thinking (Czeżowski 2000, p. 68): (a) describing and defining, (b) ordering and systematizing, (c) explaining, (d) inferring, (e) predicting, (f) proving, and (g) verifying.

Czeżowski's definition of logical culture points to crucial knowledge-gaining procedures. Those procedures are useful for evaluating our thinking and language use not only in scientific research and debate, but also in everyday discussion (Czeżowski 2000, p. 75). So, the study of fallacies built on these procedures could also focus on fallacies present in scientific discourse and fallacies in political or legal discourse.

Czeżowski's idea is a clear example of accepting a broad framework for analysing logical fallacies. As Jadacki points out, one of the central concerns of the LWS was the systematic study of various *knowledge-creative procedures*. They include: (a) verbalizing, defining, and interpreting, (b) observation, (c) inference (deduction and induction), (d) formulating problems, and (e) partition, classification, and ordering (Jadacki, 2009, pp. 98-100; see also Koszowy 2010, p. 35). On the basis of those remarks we may state that the study of fallacies within a framework of logical culture is based on a set of rules for procedures, which are evaluated as fallacious or non-fallacious. The aim of these procedures is to gain knowledge. In this sense, we are justified in naming these procedures "knowledge-gaining".[4]

In what follows I will illustrate how particular types of logical fallacies may be put within this knowledge-gaining framework of logical culture. The next three sections will discuss examples of the logico-methodological ideas which are significant for this purpose, namely (1) the conception of describing – as presented within Czeżowski's theory of analytical description (2000), (2) the procedure of inferring – as illustrated by Łuszczewska-Romahnowa's idea of pragmatic entailment (1962), and (3) the systematization of fallacies

[4] A similar idea of putting fallacies into the framework of knowledge-gaining was suggested by Hintikka (1987, p. 232).

as proposed by Kamiński (1962).[5] I will argue that all three approaches, despite the differences between them, can in fact be put in the broad framework of logical culture.

3. Czeżowski's account of description

This section discusses some ideas related to the method of analytic description proposed by Tadeusz Czeżowski (1889–1981). His main areas of inquiry covered logic, methodology of science, epistemology, ontology, and practical philosophy.

According to Czeżowski (2000, pp. 43-45), the method of analytic description consists in making "a general description of a whole class of objects on the basis of an intuitive cognition of one or several standard elements of this class" (Gumański 2000, p. 11). The general instruction for an appropriate description points to the criteria of logical correctness of description (Czeżowski 2000, p. 68):

> We say that a description is logically correct if it is faithful and not fantastic, exhaustive and not superficial, concise and not lengthy, systematic and not chaotic.

To specify this idea, Czeżowski, directly or indirectly, points to particular rules for describing and defining. Although he does not formulate these rules as a list, it can be extracted from his writings on description (2000, pp. 68-69; see also Koszowy 2004, pp. 127-128).

Rule 1: Make a proper selection of the elements of description.

This general rule is determined by distinguishing four sub-rules:

- Make your selection in such a way "as to include in the description what is important with regard to the purpose of the description while disregarding nonessential details".
- In making your selection be consistent with the context.
- Make your description by including only the relevant qualities rather than all the qualities of the described object that have been noticed.

[5] Although Kamiński was a representative of the generation which followed the LWS, his work in methodology of science may be conceived as a continuation of the tradition of the School.

- Among these relevant qualities choose the constitutive ones i.e. those which determine the whole, and usually disregard consecutive qualities which depend on and are determined by the constitutive ones.

Rule 2: Make your description in a certain justified order, "for instance in the way established by the schemes adopted in the descriptive natural sciences" or defined by the rules of a literary analysis of poetic works.

This general rule also has a further specification:

- While describing any object indicate its similarities to and differences from other familiar objects.

An example of a condensed description is defining. According to Czeżowski, the model way of making condensed description is giving a classical definition *per genus proximum et differentiam specificam*, "where by specifying *genus* we indicate similarities between the described object and those which constitute this genus with it, while *differentia* is a difference characterizing *definiendum* with the *genus*" (2000, p. 69). Here we can observe that Czeżowski's concept of description is strictly linked to the concept of definition. Defining is one of the crucial knowledge-gaining procedures, since definitions (as results of defining) are often understood as means for avoiding fallacies (see Kublikowski 2009; Koszowy 2013); for the discussion of the relationship between description and definition see also Kublikowski (2010).

The aforementioned rules of description allow us to make some remarks on the correspondence between Czeżowski's concept of description and some contemporary research strands in the study of argumentation. The use of the concepts of standards and rules by many argumentation theorists shows that the important condition for grasping the concept of fallacy is the construction of a model of procedures, within which typical fallacies are performed. "Model" is understood as a systematic set of rules. In this sense, rules of argumentative discussion constitute a model of argumentative procedures. Among various kinds of models, some authors elaborate the "ideal normative model of the speech acts performed in a critical discussion" (van Eemeren & Grootendorst 1987, p. 298) or a model of knowledge-gaining questioning procedures (Hintikka 1987, pp. 231-232). We should however bear in mind that models are not treated here as universal tools that help us recognize fallacies in a discourse. The authors who aim to establish these kinds of models do not suggest that these models should guarantee that fallacies are not committed, although we have the possibility of comparing our factual argumentative procedures with their ideal model.

The general procedure for identifying fallacies committed within knowledge-gaining procedures consists of specifying rules for these procedures. In other words, we cannot claim that a fallacy has been committed within a given procedure, unless we refer to the set of rules for the correctness of that procedure. Then we may identify a fallacy – we just compare the results of applying a given procedure with the rules. This method can be seen when applying e.g. pragma-dialectical rules of argumentation to various argumentative procedures (van Eemeren and Grootendorst 1992). A similar method is applied also to other knowledge-gaining procedures. If we deal, for example, with a questioning procedure, the result of this procedure is a question (or a set of questions). By using the rules of proper questioning, we can tell whether a given question (or a set of questions) breaks any of those rules. If at least one of the rules has been broken, a fallacy has been committed.

This context allows a list of the rules of description, based on the work of Czeżowski, to be conceived as a general model for identifying fallacies of description. For example, the list of rules of description allows us to make a list of possible fallacies, such as:

- a description which focuses on nonessential details while disregarding what is important with regard to the purpose of the description;
- a description based on a selection which is inconsistent with the context;
- a description which includes irrelevant qualities of the described object;
- a description which includes consecutive qualities (instead of constitutive ones);
- a description made without any justified order;
- a description of an object made without indicating its similarities to and differences from other familiar objects.

Although this list is just an initial attempt at proposing argument evaluation using the rules given by Czeżowski, it clearly shows, for the purpose of this paper, the idea of analysing some logical fallacies within the framework of analytic description.[6]

4. Łuszczewska-Romahnowa's pragmatic account of entailment

The main areas of research covered by Seweryna Łuszczewska-Romahnowa (1904–1978) are mathematical logic, methodology of science and seman-

[6] See Kublikowski (2009) for an account of the rules of definition and description within the structure of arguments.

tics (Batóg, 2001). One of the topics significant for the scope of this paper is the pragmatic concept of entailment (Łuszczewska-Romahnowa 1962) which has been employed in defining fallacies of reasoning, such as formal fallacies (*non sequitur*) and begging the question (*petitio prinicipii*). The theoretical framework for this inquiry is built upon the concept of argument, which is explained by the example of argument structure called Arg^+:

$$Arg^+ \begin{cases} P_1(as) \\ P_2(as) \\ P_3(P_1, P_2) \\ P_4(as) \\ P_5(P_1, P_3, P_4) \end{cases}$$

The elements of such a structure may be characterized as follows:

- proposition P_i marked with as is qualified as asserted;
- propositions $P_a, P_b, \ldots$ placed on the right from P_i qualify P_i as inferred from P_a and P_b;
- P_5 is the thesis of Arg^+.

This means that Arg^+ may be interpreted as follows: *since* P_1 *and* P_2*, then* P_3*; and since,* P_1 *and* P_3 *and* P_4*, then* P_5.

Arguments are here conceived as finite sequences of qualified propositions, i.e., those which are asserted or inferred from asserted ones. On the basis of the concept of qualified (ql) propositions, Łuszczewska-Romahnowa proposes the following definition of argumentation: a finite sequence of qualified propositions $P_1(ql_1), \ldots, P_n(ql_n)$ is called an argumentation about the thesis T if:[7]

1. the qualification ql_i of a given proposition in this sequence is either a qualification of an asserted proposition (as) or qualifies the proposition P_i as inferred from one or more propositions placed before i-s;
2. it is not the case that any sub-sequence of argumentation satisfies the conditions determined above.

In order to justify the applicability of this model in argument analysis and evaluation, Łuszczewska-Romahnowa focuses on two problems:

- how to formulate a counterplea for a given argument in order to justify the rejection of the main thesis of the argument?
- what are the possible fallacies of argumentation?

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[7] An initial condition holds that T is placed last in this sequence; this condition, however, tailors argumentation structures just to those in which the conclusion is at the end of the argumentative structure. For a broader account of argument structures see e.g. Trzęsicki (2011, pp. 61-63).

The connection between these two problems lies in the fact that a counterplea may be treated as an assessment of an argument which points to a typical fallacy. The *counterplea for the argumentation* is conceived as any proposition concerning this argumentation which may justify the refusal of the thesis of a given argument. Łuszczewska-Romahnowa lists possible counterpleas to Arg^+:

(1) P_1 is false or P_2 is false or P_4 is false.

or, equivalently:

(1') Some of the assumptions of Arg^+ are false.

The next possible counterplea states:

(2) P_1 has not been justified or P_2 not been justified or P_4 not been justified in a given theoretical context.

or, equivalently:

(2') One of assumptions of Arg^+ has not been justified.

The third counterplea may be formulated as follows:

(3) P_1, P_2 do not entail P_3 or P_1, P_3, P_4 do not entail P_5.

or, equivalently:

(3') For a given argument Arg^+, its premises do not entail its conclusion.

The next counterplea refers to assumptions of an inference:

(4) The implication $(P_1 \wedge P_2) \rightarrow P_3$ has not been justified or the implication $(P_1 \wedge P_3 \wedge P_4) \rightarrow P_5$ has not been justified.

or, equivalently:

(4') For Arg^+ the implication which is the assumption of the inference has not been justified.

These counterpleas constitute a groundwork for discussing typical fallacies of reasoning. For instance, the counterplea (2') which holds that one of the assumptions of Arg^+ has not been justified may be treated as a charge of *petitio principii*, and the counterplea (4') which states that for Arg^+ the implication which is an assumption of this inference has not been justified, is in fact the charge of *non sequitur*. The counterplea (4') is a point of departure for defining *pragmatic entailment*:

> The sequence of propositions $p_1, \ldots, p_n$ entail pragmatically the proposition p_k (given the theoretical context) iff the implication $p_1, \ldots, p_n \rightarrow p_k$ has been justified within this context.

This concept of entailment allows formulating a sentence which is equivalent to the counterplea (4'):

(4") For Arg^+ , the conclusion does not follow pragmatically (relatively to a given context) from its premises.

Amongst major objections which may be raised against the proposed approach there are:

1. the poor repertoire of logical fallacies – as this approach focuses just on general kinds of fallacies of reasoning,
2. the fact that the author considers just one direction of argumentation: from the set of premises to the set of conclusions,[8]
3. the fact that the given understanding of counterpleas represents a very narrow approach, tailored to a couple of counterpleas to a given inference;[9] however, this conception is a clear example of accepting the dialectical approach to arguments, even within such a limited model.

Despite its weak points, this general framework for analysing fallacies of reasoning is a clear example of the logical approach to arguments in the Lvov-Warsaw School – as discussed in Section 2. It also corresponds to contemporary study of argumentation. Amongst the significant similarities there are:

- the fact that the structure of arguments proposed by Łuszczewska-Romahnowa takes into account not only arguments – but also counterpleas to arguments. We can here observe the dialectical dimension of the proposed account;
- the tendency to use the description of argument structure to identify typical logical fallacies; although the structure of argument proposed by Łuszczewska-Romahnowa allows us to identify only basic logical fallacies of reasoning;
- an attempt at precisifying main claims of argumentation theory by defining pragmatic entailment and counterpleas.

Because of the fact that the approach discussed in this section is tailored to particular types of fallacies, the next section aims at answering the following question: is there any broader account of logical fallacies formulated from the viewpoint of the conceptual system of the LWS which may be employed in the contemporary study of arguments?

[8] For the inclusion of other relationships between propositions, such as the direction of entailment and the direction of justification, see the method of argument diagramming proposed by Trzęsicki (2011).

[9] For the study of the role of counterpleas within the structure of argumentation see Budzyńska (2004, p. 142) and Nieznański (2006, p. 117).

5. Kamiński's systematization of logical fallacies

A broader methodological approach to logical fallacies was proposed by Stanisław Kamiński (1919–1986). Kamiński was one of the leading researchers of the Lublin School of Philosophy at the Catholic University of Lublin. Amongst his vivid research interest in the methodology and philosophy of science (see Bronk 2001), he attempted to classify linguistic fallacies (Kamiński 1960), and typical logical errors and fallacies (Kamiński 1962). The crucial idea present in this area of Kamiński's inquiry is treating logical fallacies as a broad class of instances of human irrationality. Logical and methodological norms of carrying out knowledge-gaining procedures were, for Kamiński, the fundamental point of departure for dealing with fallacies. Since this approach is in accord with some crucial research strands in Polish analytical philosophy, I will first briefly discuss the research context, within which Kamiński's approach will be later considered.

Systematizations of logical fallacies which were built within the tradition of the Lvov-Warsaw School constituted a substantial part of academic courses in logic (see Jadacki 1997; Hołówka 2002), for knowledge of the most typical logical fallacies was treated as a necessary foundation for the acquisition of analytical skills. Moreover, knowledge and skills of recognizing logical fallacies were also claimed to be one of the key dispositions of the researcher (see e.g. Ajdukiewicz 1974). So, logical fallacies were considered in the LWS not exclusively from the point of view of teaching logic, but also from a scientific perspective. Hence, the framework for analysing fallacies in the LWS is deeply grounded in the methodology and philosophy of science. This methodological attitude is clearly exposed by Woleński (2009), who points to the philosophy of science as one of the most important areas of inquiry in the LWS. This general characteristic of the study of fallacies in Polish Analytical philosophy is deeply grounded in a framework of logical culture, which was discussed in Section 2.

The systematization of logical fallacies proposed by Kamiński is fully in line with those research tendencies. It may justifiably be treated as a unique work in Polish logical studies, for Kamiński does not deal exclusively with one particular type of logical fallacies (of description, of reasoning or of questioning), but he attempts to classify a broad class of logical fallacies. Moreover, instead of proposing an uncodified list of fallacies he puts forward a framework for analysing and classifying fallacies based on a broad conception of logic (see Koszowy 2010, pp. 32–34). Since logical fallacies are treated here as violations of norms of logic, the concept of a logical norm constitutes a point of departure for further analysis. The term "logical norm" may be understood thus:

1. in a narrow sense – as any violation of the norms of formal logic;
2. in a broad sense – as any violation of the norms of logic conceived as a discipline which encompasses: formal logic, semiotics and methodology of science.

When building his systematization of fallacies, Kamiński accepts the broad understanding of "logic" and "logical norm". Amongst the norms of logic in a broad sense there are: (1) rules governing cognitive activities (epistemology),[10] (2) rules for deductive inference (formal logic), (3) rules for language use as elaborated in semiotics (syntax, semantics and pragmatics), and (4) rules for scientific inquiry, i.e. rules of definition, questioning, rules for inductive inference (as elaborated in the methodology of science).

On the basis of this broad account of logical norms, Kamiński refers to the general understanding of "logical fallacy" as a hidden incompatibility of a given cognitive procedure with the rules of logic (1962, p. 25). Following this definition he distinguishes four general types of logical fallacies, namely:

- epistemological fallacies;
- semiotic fallacies;
- fallacies of reasoning also labelled "logical fallacies in a strict sense";
- methodological fallacies of applying rules governing knowledge-gaining procedures.

Basing on this general distinction, Kamiński proposes the following systematization of logical fallacies:[11]

A. Epistemological fallacies

I. Improper cognition of a given subject-matter

1. Cognition directed by extra-cognitive factors:
   (a) superstitions which are the effect of education, social environment or laziness of thought;
   (b) cognition dominated by affects (emotions);

[10] Including epistemology in this broad approach to logical fallacies alludes to the understanding of epistemology as part of logic broadly conceived. A historical illustration of this idea is the labelling of epistemology as *logica maior* (or *logica materialis*), as distinguished from *logica minor* (or *logica formalis*), i.e. formal logic.

[11] For the purpose of the paper I tailor Kamiński's systematization just to the main types of logical fallacies. For the details of this account see Kamiński (1962, pp. 29–39).

   (c) inclinations towards: hasty generalization, anthropomorphisation, oversimplification and not distinguishing between the main domains and methods of cognition;
2. Cognition of the formal subject (i.e. aspect) unsuitable for a given cognitive disposition.
3. Cognition gained without proper coordination of the various cognitive dispositions and their mutual control;
4. Statements based on careless or inexact perception;
5. Statements which extend the act of perceiving;
6. Statements determined by alleged obviousness, especially when they are not preceded by a sufficient analysis of facts.

II. Uncritical reception of information

1. Information gained without any critical cognitive activity:
   (a) gained in the process of education, readings, tradition, social environment, habits, and superstitions;
   (b) accepted automatically only because of the fact that the new information is coherent with one's system of beliefs.
2. Information gained only because of the irrational acceptance of authority:
   (a) because of the association of the source of information with the famous name of someone, who is not in fact an authority in a given domain;
   (b) because of social or material profit associated with accepting this information.
3. Information accepted only because of the form of utterances:
   (a) a convincing formulation of information in the form of a paradox or a slogan;
   (b) information which is repeated (e.g., advertisement or propaganda).
4. Information gained from:
   (a) inadequate sources (e.g. indirect sources instead of direct sources);
   (b) inauthentic sources.

B. Semiotic fallacies

I. Insufficient level of understanding of an utterance caused by:

1. The use of professional terminology of a given field or the use of untypical metaphors or other specific expressions;

2. Invalid syntax and structure of a complex utterance;
3. Non-uniform intension of an utterance caused by the use of too general expressions, in particular:
   (a) confusing languages of different levels;
   (b) confusing various functions of language, e.g., informative, evaluative, expressive, and evocative.

II. Ambiguity of an utterance:

1. The use of an ambiguous phrase where the context does not enable one to determine exactly one meaning;
2. Equivocation in reasoning (the fallacy of four terms);
3. The unambiguous use of expressions, which in fact have various meanings that should be distinguished;
4. Amphiboly – the ambiguous structure of an utterance;
5. Informational shortcuts which cause misunderstanding;
6. Partial ambiguity of an utterance.

III. Unclear understanding of an utterance

1. Undetermined extension (vagueness);
2. Undetermined intension;
3. Insufficiently precise formulation of a complex utterance.

C. Fallacies of reasoning (logical fallacies in a strict sense)

I. Apparent indirect justification

1. Persuasion aimed at the direct assertion of a statement.
2. Forcing the assertion of a thesis upon someone.
3. Making someone assert a given proposition by employing *argumentum ad personam.*
4. Forcing the assertion of a thesis by hiding or modifying this thesis.

II. Invalid reasoning

1. *Ignoratio elenchi* – ignorance of the thesis which is being argued for.
2. Modifications of the thesis which is being argued for.
3. Apparent conclusiveness of reasoning.

D. Methodological fallacies

I. General methodological fallacies

1. Fallacies of defining:
   (a) formal;
   (b) informal;
2. Fallacies of division and classification:
   (a) formal;
   (b) informal;
   (c) semantic.
3. Fallacies of posing and solving scientific problems:
   (a) invalid formulation of questions and hypotheses;
   (b) invalid solutions of problems.
4. Fallacies of discussion:
   (a) improper choice of the subject-matter of a discussion;
   (b) improper point of departure of a discussion, e.g. posing a problem which is not relevant to the main controversy or posing a problem which is undecidable or intractable as decidable or tractable;
   (c) employing semiotic fallacies in deception;
   (d) convincing by means of emotional actions;
   (e) employing eristic fallacies;
   (f) employing invalid inferences.

II. Specific methodological fallacies

1. Fallacious use of the formal-deductive method of scientific inquiry.
2. Fallacious use of the statistical method.
3. Fallacious use of the empirical method.
4. Fallacious use of the historical method.
5. Fallacious use of the methods of philological analysis.[12]

The systematization of logical fallacies proposed by Kamiński seems to capture a fairly rich repertoire of fallacies. It also clearly points to some specific rules (of epistemology, semiotics, formal logic and the methodology of science) useful for assessing utterances.

[12] Kamiński states that within this subsection the fallacies of criticizing texts and fallacies of reasoning may be repeated.

However, this approach to systematizing logical fallacies raises serious objections. At first glance, Kamiński's broad account of logical fallacies pays the price for its imprecision: providing such a rich systematization of fallacies results in vague distinctions. In fact, two major objections to this systematization may be raised:

- this conception is too broad because it covers fallacies that are not violations of any logical norms strictly understood; for instance, it would be very difficult to point to any logical norm, strictly understood, which would be violated in the case of improper measurement – which is considered by Kamiński as a specific methodological fallacy (Kamiński 1962, p. 38);
- the main types of fallacies overlap; for example, the fallacy *post hoc ergo propter hoc* may be classified both as a fallacy of reasoning and as a methodological fallacy; the fallacy of four terms (*quaternio terminorum*) may be classified both as a fallacy of reasoning and as a semiotic fallacy; moreover, affirming the consequent may be classified as a fallacy of reasoning, amphibology as a semiotic fallacy and the vicious circle of definition as a methodological fallacy; Kamiński is fully aware of the overlap between the main types of fallacies, for he indicates some fallacies which in fact belong to different types (p. 39).

Kamiński is also aware of a number of other difficulties with building an adequate systematization of fallacies. Amongst them he mentions modifications of the very notion of logic (Kamiński 1962, pp. 25–26). He also points out that many historical attempts at classifying logical fallacies take into account the broad class of coginitive fallacies, of which the so-called "logical fallacies" constitute one subclass.

Despite these and other objections, this representation clearly shows how logical fallacies may be put into the conceptual system of the LWS. Kamiński observes that despite its defects, the proposed codification of logical fallacies plays an important cognitive role, i.e. it helps deepen understanding of rational thinking and cognition (p. 26). In this respect, Kamiński's approach to logical fallacies differs from the pessimistic opinion of the study of fallacies as cognitively useless (see e.g. Lambert and Ulrich 1980). According to Kamiński, even though many attempts at classifying fallacies are imprecise and fail to capture some significant fallacies, they give a theoretical framework useful for ordering common cognitive mishaps and failures, thus sharpening the tools for identifying them. Hence, the tradition of Polish analytical philosophy may be placed amongst "optimistic" accounts of fallacies. Moreover, this optimistic attitude is supported by precise methodological tools useful in identifying fallacies. In this context, Kamiński's systematization may be conceived as a unifying account which aims to grasp a variety of violations of the rules of proper cognition.

Thus, Kamiński's systematization clearly illustrates the accepted model of a critical thinker, who may be conceived as a person who avoids typical violations of logical norms. For example, the proposed model is in accord with the skills of a critical thinker who is characterized by informal logicians (e.g., Hoaglund, 2002, pp. 5-6) as a person who is well-informed, fair-minded in evaluation, who can distinguish fact from opinion and reliable reports from erroneous ones. These similarities point to a significant topic for further inquiry, which would be to discuss systematically the ideal of a critical thinker in the tradition of the LWS, as compared to the Critical Thinking Movement in North America.

6. Towards a programme for the study of fallacies

In the concluding section I suggest some future research tasks which may be accomplished within the research programme of the study of fallacies rooted in the conceptual system of the LWS. The three ideas discussed (proposed by Czeżowski, Łuszczewska-Romahnowa, and Kamiński) point to research fields which may constitute the focus of a future project. The ideas discussed in the paper show that in order to analyze typical fallacies we should establish sets of rules (models) for particular knowledge-gaining procedures. On the one hand, those procedures are employed within argumentative discourse, so they are evaluated within argumentation theory. On the other hand, the same kinds of procedures are crucial in scientific research, so they are investigated by the methodology of science (see Koszowy 2013). The procedural rules for the argumentative process are built either in argumentation theory (van Eemeren and Houtlosser 2002, pp. 14-15) or in the methodology of science (Czeżowski 2000, p. 51). Hence, the methodology of science can serve not only as a source of inspiration for fallacy theorists, but also as a foundation for fallacy theory, since it provides well-developed tools for recognizing fallacies.

Moreover, the paper shows that the study of fallacies in the LWS was far from the imprecise fallacy approach, and much closer to detailed fallacy analysis. The foundation for such an approach lies in:

- the broad concept of a logical norm,
- the methodological approach to logical fallacies conceived as violations of the rules of performing knowledge-gaining procedures.

Another concluding remark is that within the conceptual system of the LWS there are definitely no attempts at building a fallacy theory as such. In other words, there is no urge to build a general theory of fallacies.[13] The reason for this attitude is the fact that the concept of a fallacy in the LWS is related to the concept of logical norms broadly conceived. Thus, within the tradition of Polish analytical philosophy there is definitely no need to build a separate theoretical framework for the study of fallacies.

Moreover, the conceptual system of the LWS contains an idea which is most significant for our purpose of grasping the concept of a logical fallacy. The tendency of analysing logical fallacies within a broad logical framework is in accord with major contemporary tendencies in the study of fallacies. In early works in argumentation theory a dominant tendency was to build a fallacy theory (Johnson 1987). However, some argumentation theorists and informal logicians (e.g. Johnson, 1987; van Eemeren, 2001; Hansen 2002) have noted the major difficulties of building such a theory. Those difficulties were among the reasons for rejecting the urge to theorise in the study of fallacies. Amongst many alternative propositions is the approach proposed by Cummings (2004, pp. 90-91) who argues for focusing on a descriptive approach to the study of fallacies which is to be founded on the analysis of relationships between the crucial concepts of argumentation theory. Within this context, the study of fallacies from the point of view of the conceptual system of the LWS appears to be a methodological programme close to such descriptive approaches developed by some logicians and argumentation theorists who pursue the analysis of concrete fallacies without stressing the need to build a fallacy theory (see e.g. Jacquette 2009).

As the discussed examples show, a strength of the programme of the study of fallacies based on the conceptual system of the LWS lies in the methodological foundations of the study of fallacies. This programme is close to criterial approaches to fallacies. Hence, it clearly differs from the unclear *fallacy approach*, which was present in argumentation theory at its earlier stage. In seeking possible directions for further research, the conceptual system of the LWS may serve as a framework for analysing and assessing various knowledge-gaining procedures.

[13] Some arguments which question the need to build a unified theory are given or discussed by Hamblin (1970), Hintikka (1987), Johnson (1987), and Hansen (2002).

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HOW TO ASSESS THE VALUE OF AN ACT?

Abstract. The aim of this paper is to reduce evaluation of acts to evaluation of events. To achieve this goal we explicate the notion of act used in law and ethics in terms inspired by the symbolism of Wolniewicz's ontology of situations and present some consequences of such explication. In particular, bad acts are distinguished from wrong acts. Two notions of wrong act are introduced: the internally wrong act and the externally wrong act.

1. Values and responsibility: moral and legal

The question whether values are of objective or subjective character is one of the basic issues of ethics.

If one admits that values are of a subjective character, one must also admit relativism and consequently deny the possibility of objective attribution of responsibility based on values, *i.e.* based on the principle of justice. In a such case, responsibility for any act could by attributed objectively only when a certain rule is broken by the act. However, attribution of responsibility in connection with breaking rules is relative by definition, provided that rules have no objective justification.

In ethics, the question whether values are of objective character has two faces:

- the first is the question of the objectivity of values "as such" and
- the second is the question of the possibility of the objective assessment of acts.

Values "as such" can be recognized easily. Moreover, one can easily admit that the important part of them have an objective character (*e.g.* values "as such" like life, safeness, healthiness, family, human society, civilization) and constitute a hierarchy which is also of an objective character. However, it is much more difficult to learn the values of acts. In fact, we have no clear notion of an act as well as any clear criterion for the evaluation of acts.

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The question whether values are of an objective or subjective character is also important in law and jurisprudence since in some circumstances the law itself admits the superiority of values over the letter of the law, *i.e.*, it allows us, provided some conditions are fulfilled, to abandon the strict meaning of legal prescriptions to keep a legal solution fair and just. For example, Polish civil law enables the judge to refuse to provide legal protection to formally legal acts which contradict the so-called "principles of social coexistence".

Having all the above in mind, one can see that a clear notion of an act is of real importance both to ethics as well as to law and jurisprudence. In particular, such a notion should make easier the evaluation of acts.

2. Ontology of situations: basic ideas

In the next paragraph, we will use some basic ideas of Wolniewicz's ontology of situations, combined with some ideas of temporal logic and possible worlds semantics, to clarify the notion of act. Therefore, let us recall a few basic ideas of Wolniewicz's ontology here.

Professor Wolniewicz considers a structure $\langle SE, \leq \rangle$, where SE is a set of so-called "elementary situations" and $\leq$ is a partial order. An elementary situation can be perceived as a semantic correlate of an elementary conjunction in Wolniewicz's sense (*i.e.* can be perceived as a semantic correlate of a simple sentence or a conjunction of simple sentences). For SE holds:

$$SE = SE'' U \{o, \lambda\}$$

where SE'' is a set of proper elementary situations, o ("zero") is an empty situation and λ is an impossible situation. For any elementary situation x holds:

$$o \leq x \leq \lambda.$$

For any elementary situations x and y holds:

$$x \ ; y = y \Leftrightarrow x \leq y \Leftrightarrow x!y = x.$$

The set SP is the set of maximal elementary situations (or "possible worlds"). The set SA is the set of atoms (*i.e.* $SA = \{x \in SE : x \text{ covers } o\}$). For atoms the following relation is defined:

$$x, y \in SA \Rightarrow (x \approx_d y \Leftrightarrow (x = y \vee x \ ; y = \lambda)).$$

If the set SE'' is not empty, $D = SA/{\approx_d}$ is a set of logical dimensions of SP. Wolniewicz's assumption is that the number of logical dimensions is finite.

In Wolniewicz's ontology, every possible world has exactly one atom from each dimension.

3. Alternative events: acts and choices

Let S be the set of Wolniewicz's structures described above (*i.e.* $S = \{s_i : s_i = \langle SE_i, \leq \rangle\}$) and T be a linearly ordered set of time moments. For any $t \in T$ we attribute exactly one structure $s \in S$. As a result some structures are linearly ordered. Respectively, some sets of possible worlds are linearly ordered (we may number them $SP_1, SP_2, SP_3, \ldots$).

In the set $\{SP_1 \cup SP_2 \cup SP_3 \cup \ldots)$ we define a relation of attainability R. For R we assume only that:

$$xRy \Rightarrow \text{there are } SP_n \text{ and } SP_{n+1} (x \in SP_n \text{ and } y \in SP_{n+1}).$$

Any act is represented by a pair of elementary situations $\langle \boldsymbol{a}, \boldsymbol{b} \rangle$ such that:

(1) if situation $\boldsymbol{a}$ has an atom from a certain dimension then $\boldsymbol{b}$ has an atom from the same dimension and *vice versa*, *i.e.* for any D_i:

$$W(D_i, \boldsymbol{a}) \Leftrightarrow W(D_i, \boldsymbol{b})$$

(we read "$W(D_i, \boldsymbol{a})$": "the situation $\boldsymbol{a}$ has an atom from D_i"; intuitively, according to this condition $\boldsymbol{a}$ and $\boldsymbol{b}$ are alternative arrangements of a certain fragment of the world),

(2) $\boldsymbol{b}$ is attainable from $\boldsymbol{a}$, *i.e.* there are $x \in SP_n$ and $y \in SP_{n+1}$ such that:

$$\boldsymbol{a} \leq x \text{ and } \boldsymbol{b} \leq y \text{ and } xRy$$

(3) $\boldsymbol{b}$ is not a necessary consequence of $\boldsymbol{a}$ (intuitively, $\boldsymbol{b}$ is the result of a choice), *i.e.* there is $z \in SP_{n+1}$ such that:

$$xRz \text{ and}$$

for any atoms $\boldsymbol{k}$ and $\boldsymbol{m}$ such that $\boldsymbol{k} \leq y$ and $\boldsymbol{m} \leq z$ and for any dimension D_i:

$$(W(D_i, \boldsymbol{a}) \text{ and } \boldsymbol{k} \in D_i \text{ and } \boldsymbol{m} \in D_i \Rightarrow \boldsymbol{k} \,;\, \boldsymbol{m} = \lambda)$$

and

$$(-W(D_i, \boldsymbol{a}) \text{ and } \boldsymbol{k} \in D_i \text{ and } \boldsymbol{m} \in D_i \Rightarrow \boldsymbol{k} = \boldsymbol{m}).$$

Intuitively, according to the conditions defined above, any act is a choice between alternative events, where alternative events may be perceived as the alternative ways in which a person in a choice situation can arrange a certain fragment of the world. In this sense, every choice is an act and every act is a choice. Respectively, if one choose to do nothing, doing nothing is acting. Further, if one has no choice, there is no acting.

4. Consequences

Having the above explication of the notion of act in terms of a choice between alternative events, we can reduce the evaluation of acts to an evaluation of events.

If a choice is between exactly two alternative events, which both are atoms in Wolniewicz's sense, we will call this choice an "elementary choice".

Let us define now the notion of the internal value of an act and the notion of the external value of an act. The internal value of an act is the difference between the value of the chosen alternative event and the value of the best of the rest of the alternative events in a given choice situation. Respectively, the external value of an act is the difference between the value of the chosen alternative event and its consequences, and the value of the best of the rest of the alternative events and their consequences, in a given choice situation. In this way, the evaluation of acts is reduced to comparing the values of situations.

Having in mind the above, one can define the notions of a bad act, an internally wrong act and an externally wrong act in the following way:

1) a bad act is any act with negative internal value in an elementary choice situation,
2) an internally wrong act is any act with negative internal value,
3) an externally wrong act is any act with negative external value.

Respectively, *e.g.* killing people always is bad (since, in an elementary choice situation "to kill or not to kill" killing has always a negative internal value) but sometimes may be not wrong (since in a non-elementary choice situation "to kill the terrorist or not to kill him" killing may have positive internal value as well as it may have positive external value).

Among these three notions, the notion of an externally wrong act seems to be the most suitable criterion for evaluation of the acts. This is because if one admits as the criterion in question the internal value of an act, one consequently arbitrarily excludes from assessment all consequences of alternative events.

Respectively, the possibility of the objective assessment of acts seems to depend on the possibility of objective deciding on the external values of acts, in the sense defined above.

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SOME PROBLEMS IN REPRESENTATION OF KNOWLEDGE IN FORMAL LANGUAGES

Abstract. In the article we discuss the basic difficulties which we have when we use a formal language to describe knowledge of an agent. In particular we discuss the difficulties if we accept positive and negative introspection of an agent, Fitch's paradox, the problem of logical omniscience and some ways to avoid them.

Keywords: knowledge, epistemic logic, omniscience, Fitch's paradox, introspection.

Some of the axioms of $S5$ epistemic logic are controversial, or lead to results that are regarded in some cases as undesirable. We will now describe the controversy and some problems that have been considered in the case of some axioms of modal epistemic logic.

Negative introspection of an agent

Real agents have limited cognitive self-awareness of knowledge. Depending on the type of agent, the agent may know that he knows something or does not know it. By accepting axioms which express the introspection of an agent we also accept the self-reflexivity of knowledge of this agent. However, the self-reflexivity of knowledge of the agent is at the core of various paradoxes.

Let us consider, for example, the difficulties which we have if we describe the knowledge of an agent who has knowledge about only one atomic sentence p. We assume that the agent is the ideal cognitive agent and he knows all the logical consequences of his knowledge (in the discussed case, the agent knows all the logical consequences of the sentence p) and he can be introspective of his knowledge. The question is whether knowledge about p and the logical consequences from p constitues the whole knowledge of our

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agent. Intuitively, it seems that the proper answer is *yes.* Let us assume that q is an atomic sentence different from p. The agent, in accordance with the accepted premise in the introduction, has knowledge about p only and he has no knowledge about q. So, we can conclude that $\neg Kq$. We have assumed, however, that the agent is the ideal cognitive agent and can be introspective of his knowledge. Thus, if the agent has knowledge of his ignorance about the sentence q, then $K\neg qK$. Although we have assumed that the agent has knowledge only about p, he also knows that $\neg Kq$. But $\neg Kq$ is not a logical consequence of p.

We have a more interesting case if we consider more than one agent. To distinguish "old" agent and "new" agent, let us assign the indices 1 and 2 respectively. Since by our assumption, agent 1 has only knowledge about p, so it is true that $\neg K_1q$. Agents have knowledge only about true sentences. Therefore, since the sentence K_1q is false, agent 2 does not know that K_1q, then $\neg K_2K_1q$. Agent 1 knows that agent 2 does not know that agent 1 knows that q, so $K_1\neg K_2K_1p$. Agent 2 can also be introspective of his knowledge. So it is true that agent 1 knows that agent 2 knows that agent 2 does not know that agent 1 knows that q. Formally, we can write this as follows: $K_1K_2\neg K_2K_1p$. Agent 1, although his knowledge was limited to p, knows a nontrivial fact about the knowledge of agent 2.

The difficulties that have arisen in the considered example, are due to the negative introspection of agents. If we reject the negative introspection of agents, the model which we have used to characterize the knowledge becomes simpler and free of some contradictions.

Fitch's paradox

Fitch in [4] argued that if there are unknown truths, then there are also unknowable truths. The argument presented by Fitch sparked a lively and long discussion among philosophers and epistemologists. We will present a reconstruction of the reasoning described by Fitch.

Allow that p is true and unknown. We can write this as $p \wedge \neg Kp$. If $K(p \wedge \neg Kp)$, then if we use for it a rule $K(\varphi \wedge \psi) \vdash (K\varphi \wedge K\psi)$, we get $Kp \wedge K\neg Kp$. So, the sentences Kp and $K\neg Kp$ are true. However, using for the second sentence a rule $K\varphi \vdash \varphi$ we get $\neg Kp$. We have shown that under the assumption that $K(p \wedge \neg Kp)$ the sentences Kp and $\neg Kp$ are inferable. It can not be that $K(p \wedge \neg Kp)$, because this leads to a contradiction.

Let us assume that $\neg K(p \wedge \neg Kp)$. If we apply Gödels's rule to this formula we obtain $\Box\neg K(p\wedge\neg Kp)$. The operators of necessity and possibility are definable for each other ($\Box p \equiv \neg\Diamond\neg P$) so we get $\neg\Diamond\neg\neg K(p \wedge \neg Kp)$. Hence, from the double negation law and rules of substitution we get:

$$\neg\Diamond K(p \wedge \neg Kp).$$

The above formula says that it is impossible to know that $p \wedge \neg Kp$. So, if p is true and unknown, then the fact that p is true and unknown is unknowable.

If we accept the principle that *if something is true, then it is possible to know it* ($\varphi \rightarrow \Diamond K\varphi$), then if we assume that $p \wedge \neg Kp$, from the above principle and Modus Ponens we get:

$$\Diamond(p \wedge \neg Kp).$$

This is in contradiction with the previously obtained formula. Since the assumption that $p \wedge \neg Kp$ leads to a contradiction (regardless of whether we believe that this formula is known or unknown), so we assume that $\neg(p \wedge \neg Kp)$. From the last formula and the thesis of classical logic, we have that $p \rightarrow Kp$, which can be interpreted as: if p is true, then p is known. The assumption that $p \wedge \neg Kp$ and the commonly accepted rules leads to a contradiction. The rejection of this assumption leads to the conclusion that if something is true, it is known. We made no assumption on p, so from the above considerations we conclude that all truths are known. This is, of course unintuitive and inconsistent with reality. We do not know all truths and we have not been bestowed with the property of omniscience.

The above reconstruction of Fitch's argument could not be carried out if we had not established the acceptability of two rules, essential for our reconstruction: the rule of the distributivity of the knowledge operator with respect to the conjunction operator $K(\varphi \wedge \psi) \vdash (K\varphi \wedge K\psi)$ and the rule of the factuality of knowledge $K\varphi \vdash \varphi$. It seems that the rejection of these two rules is sufficient to ensure that Fitch's argument can not be reconstructed. Such an approach, however, is an unsatisfactory approach for a few reasons. Firstly, both rules have good reasons in knowledge systems. The rule of factuality of knowledge has been widely discussed as a prerequisite of knowledge. The rule of distributivity is quite intuitive and if we reject this rule we obtain a notion of knowledge with very specific properties. Secondly, the rules are not necessary for the reconstruction of Fitch's argument. Tennant in [12] and [13] shows that it is possible to reconstruct Fitch's argument without involving the rule of factuality of knowledge. Williamson in [15] formulate a version of the principle of intelligibility such that it is possible to derive inconsistent consequences without refering to the distribution of knowledge operator with respect to the conjunction operator[1].

Omniscience problem

The best known way to formalize knowledge and belief is an epistemic modal logic. Modal languages are a combination of the high power of expression and intuitive semantics – the reason they are a great tool used in artificial intelligence. The main achievement of modal logics is the transformation of extensional languages to intensional languages.

Unfortunately epistemic modal logic suffers from an ailment called *logical omniscience*. The problem of logical omniscience makes formal systems which use epistemic logic not a proper tool for modeling real agents, because real agents are never logical omniscient. The basic axiom of normal modal logics $K(\varphi \to \psi) \to (K\varphi \to K\psi)$ leads to the conclusion that agents knows all logical consequences of their knowledge. This assumption is unreasonable in relation to the real agents (eg. people) or agents with bounded memory (eg. processors in computer systems). If we assume, for example, that an agent knows the algorithm of how to decompose a number into its prime factors, then the agent with common assumptions being made in the formal system, used to describe the knowledge, should to know the decomposition into its prime factors of all natural numbers. It's hard to defend that assumption with respect to the real agents because we do not expect that a real person has a knowledge of the decomposition into prime factors of all natural numbers, which might have, for example, five hundred thousand digits. Modal knowledge operators are regarded as an idealization of knowledge operators used in reasoning by humans.

In order to reject the disputed ownership of agents, such as their logical omniscience, there are some different modifications of systems of modal logic.

Konolige's suggestion

One of the possible methods used to reject the logical omniscience of agents is the syntactic approach, in which is assumed that the knowledge of agents is represented by an arbitrary set of formulas [9], [3]. Of course, the set

[1] Fitch's argument was also discussed on the basis of intuitionistic logic [14] and [1].

we are talking about should be constructed in such a way that it is closed on the logical consequence and should include the set of all substitutions of all axioms. The notion of knowledge formulated above does not lead to the problem of logical omniscience paradox, however, knowledge understood in this way is very difficult to analyze. If knowledge is represented by an arbitrary set of formulas, we do not have any guidance on how to analyze such knowledge. Konolige [6] gives a proposal for constructing such a set. In Konolige's approach, knowledge of an agent is represented by a set of primitive facts, which is closed to the rules of inference. In this case, the problem of logical omniscience is solved by the adoption of an incomplete set of inference rules.

Montague's semantics

Montague in [8] gives possible worlds semantics for epistemic logic, in which formulas are associated with sets of possible worlds, but knowledge is not modeled as a relation between possible worlds [16]. There is no relation of accessibility in Montague's semantics. In this approach we lose the intuition that the agent knows that φ, if φ is true in all worlds considered by the agent as possible. However, in this approach we can avoid the problem of logical omniscience. There is some difficulty in Montague's semantics. Namely, in this semantics, agents do not know all the logical consequences of their knowledge, so they are not able to identify formulas which are logically equivalent.

Impossible worlds

There have been efforts of trying to avoid the problem of logical omniscience by enriching the standard semantics of possible worlds with impossible worlds. An impossible world is a world in which the proved formula may not necessarily be true, or a world in which true formulas are contradictory. Impossible worlds are worlds that only enrich the epistemic set of epistemic alternatives, however, they are not logically possible worlds. With this approach, agents may not know all the tautologies of classical logic, since there may be possible worlds, considered by the agents as possible, in which some tautologies are not true.

The proposal of a semantics which contains impossible worlds was given by Levesque in [7]. Levesque makes a distinction between *explicit knowledge*

(knowledge provided *outside*) and *implicit knowledge* (knowledge that consists of all the logical consequences that can be derived from explicit knowledge). Levesque considered a model of possible worlds in which atomic sentences can be true, false, true and false at the same time, and such that they have no specified property. Levesque gives a logic of *internal* (implicit) and *external* (explicit) beliefs. Levesque's results can be extended to the case of knowledge [5, p. 8]. Explicit beliefs imply an implicit belief, but not vice versa. In Levesque's logic there is no problem of logical omniscience because in this logic there is no rule: if φ is a tautology, then there is $B\varphi$[2].

Semantics, which allow the existence of impossible worlds are also described for example in [2], [10], [11].

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STUDIES IN LOGIC, GRAMMAR AND RHETORIC 30 (43) 2012

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SCIENTIFIC ARGUMENTATION AND THE VALIDITY OF RESULTS

Abstract. It might seem that in exact sciences, in particular in mathematics, there is nothing interesting to say about Argumentation Theory. We have a well defined language of logic, several rules and we can expect the results following these rules. However, if the context of investigations is not well known, if the language we choose is completely new, even following well established rules, we gain some conclusions, which are not easily accepted. Hence, we can pose a question: which arguments are necessary to convince others of the obtained results. In this paper we would like to focus on this problem taking as an example natural numbers.

Keywords: scientific argumentation, truth, language, relations, natural numbers

1. Introduction

Argumentation theory involves different aspects and fields of reasoning. If we think of philosophical argumentations they are distinguished because of their premises and rules of inference. The other type of argumentation – negotiations – set a goal to achieve a financial success. Every day negotiations, instead, are often hasty and mistaken because of their generality; in oral conversations, humans often fail at simple logical tasks, committing mistakes in probabilistic reasoning [4], [20]. Perhaps, this is because, as some authors state, inferential processes carried out by mechanisms for reasoning, are unconscious [12]. We cannot talk about a logical relationship between premises and conclusions.

The field where logic is very important is science. We apply a kind of argumentation consisting of true expressions. We base our argumentation on logical derivation, on the well established inferential mechanism. However, as we will show, sometimes logic is not strong enough to convince others of the obtained results.

To exemplify what we have said above, we will limit ourselves to the field of mathematics and show that to prove any kind of mathematical

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truth, we can choose any system of axioms, under the condition that it is consistent with established mathematical truths, and following rules of inference, we obtain results that necessarily follow them. Surprisingly, it does not matter whether the system is abstract or not, whether there is or there is not any relationship to classical theory. The point is that if the system is not inconsistent, we cannot deny either the truth or the validity of the results. Hence, we have no arguments to abolish a theory and we are not convinced enough to approve it. In such a case, what can be done? It has happened many times in science that an application of a theory was found much later, as in the case of imaginary numbers. The situation is different in experimental sciences, like physics. The first argument to accept a theory is its application, we have to verify it by experiment, but in mathematics – in an abstract field of reasoning, we cannot use this argument.

Hence, in the following paper, we will apply the example of natural numbers to follow the investigated line of reasoning. We will explicate Frege's idea of what a natural number is. Next, we will present two classical approaches how natural numbers can be introduced. In Section 4 we will propose a totally new language based on a concept of relation and apply it in order to define natural numbers. Section 5 will contain the formal proof of classical truths in this new relational framework. We will conclude with a discussion of results.

2. Frege's Concept of Natural Numbers

Numbers, according to Frege [6], (§45) are not obtained from objects by abstraction; they have nothing physical, nothing subjective; they are not a representation. Numbers do not take origin from the union of one object to another. Numbers, according to Frege, refer to concepts. Frege defines them by the use of the statement "the concept F is equinumerous with the concept G".

The definition of the concept of "**equinumerous**" states that there is a one-to-one correspondence between the objects which fall under F and the objects which fall under G if and only if every object falling under F can be paired with a unique and distinct object falling under G and, under this pairing, every object falling under G gets paired with some unique and distinct object falling under F. A one-to-one correspondence (i.e. function) takes objects as arguments and maps these arguments to values. With a function Frege associates the concept of "course-of-values" which is a record of the value of the function for each argument, it is also called **the extension**

of a concept F (objects which fall under the concept F), known as *Frege's Basic Law V.*

Next, Frege describes the meaning of the statement: **the number of a concept F**', which is defined as the extension or set of all concepts that are equinumerous with F. In this way the number 0 is identified with the number of the concept of not being self-identical (§74), i.e. that nothing falls under this concept. The number 1 is identified with the class of all concepts for which exactly one thing falls, etc.

Continuing, to define natural numbers Frege introduces the relational concept **n follows immediately m** (§76) in the sequence of natural numbers if and only if:

1. there is a concept F and an object x such that: x falls under F,
2. n is the (cardinal) number which falls under the concept F,
3. m is the (cardinal) number which falls under the concept of the "object other than x falling under F".

At this point Frege does not write $n = m + 1$ because the equivalence relation will qualify $(m + 1)$ as an object, but according to him, a number is a concept and not an object.

Successively, he introduces the relation of successor in a sequence (§79) – "**y follows x** in φ-sequence" as follows: "for every concept F, y always falls under the concept F," if and only if:

1. every object for which x is in relation φ with it, falls under F,
2. for any object d, if an object d falls under F, every object for which d is in relation φ with it, also falls under F.

Formally, one can say that Frege introduces the relation $x < y$ in a sequence of natural numbers (in φ sequence). Precisely, the relation φ, for Frege, is a relation which is not necessarily thought of as a spatial or time order, even though this is not excluded.

Finally, Frege introduces a **sequence of natural numbers** and defines a **finite natural number**. We will stop here in Frege's investigations because only the main idea, what a natural number is, will be useful for our purpose in this paper.

3. Classical Constructions of Natural Numbers

The 20$^{\text{th}}$ century has witnessed several attempts to build mathematics on different grounds, not only those provided by classical logic. In non-classical logical frameworks, we have different systems describing the way of introducing natural numbers and as a consequence building arithmetic, such as the intuitionistic mathematics of Brouwer, Heyting's arithmetic, Church's arithmetic, etc. [1], [2], [18].

In this section I would like to present very briefly two classical approaches, in particular Peano's axiomatization and Von Neumann's construction of natural numbers.

3.1. Peano's Axiomatization

In the axiomatic system of natural numbers of Giuseppe Peano [15] we have three primitive notions: *a concept of natural number*, *a zero element* – 0, and a function called *a successor function which gives for n its successor* – n'. Hence, one can notice that Peano's idea is very close to Frege's intuition of natural numbers. Five axioms describe in a very simple and elegant way their properties.

Let us consider N as a set of natural numbers:

Axiom 1. $0 \in N$.

Axiom 2. $n \in N \longrightarrow n' \in N$.

Axiom 3. $n \in N \longrightarrow n' \neq 0$.

Axiom 4. $m, n \in N \wedge m' = n' \longrightarrow m = n$.

Axiom 5. $(Z \subseteq N \wedge 0 \in Z \wedge \forall k \in Z k' \in Z) \longrightarrow Z = N$.

Axiom 1 states that 0 is a natural number. Axiom 2 assures that every natural number has a successor and Axiom 3 that there is no model composed only of one natural number.

Similarly, Axiom 4 with Axiom 3 assure that we cannot create a model in which only a finite number of elements would be considered as natural numbers. The successor function maps in a bijective way (one-to-one) a set of natural numbers into its part. A one-to-one correspondence is assured by Axiom 4 and again we have a parallel to Frege's equinumerous concept. Due to Axiom 3 this one-to-one correspondence is into a proper part, because nothing has a successor equal to 0. Thus, the set of elements which fulfils Axioms 1–4 has to be equinumerous with its proper part, so it has to be infinite. If we threw out Axiom 4 we could construct the following model: 0 and 1 are natural numbers and 1 is a successor of 0 and 1 is a successor of 1, as well.

Finally, the *Induction Axiom* 5 assures that in the ordinal interpretation we cannot create a model including "infinite" natural numbers which is in agreement with Frege's idea of an infinite natural number.

There are various models of Peano's arithmetic [19], for example, let us call natural numbers all even numbers: $0, 2, 4, 6, 8, \ldots$. 0 will be considered as a *zero* element and $n + 2$ – a successor of n. It is easy to verify that this model satisfies Peano's Axioms.

3.2. Von Neumann's Construction

Another construction of natural numbers was proposed by John von Neumann. Von Neumann [21] uses two primitive notions, exactly the same as those used in the axiomatization of the theory of sets of Zermelo-Fraenkel [9], [7], [17]: the *relation of being an element* – $\in$ and the concept of *set.*

The successor function of Von Neumann is defined in the following way:

Definition 1

For every set X: $X' = X \cup \{X\}$.

which is a union of two sets: X and a singleton $\{X\}$. Such a set is called: *successor*. Additionally, Von Neuman states that there exists a null set – $\emptyset$ and for every set there exists its successor.

Theorem 1

There exists exactly one family of sets N with the following properties [9]:

(1) $\emptyset \in N$,

(2) $X \in N \longrightarrow X' \in N$,

(3) if K satisfies (1) and (2), then $N \subset K$.

Hence, Von Neumann's set of natural numbers is composed of the following elements: $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}, \}, ...\}$. Every element of this set defines a natural number: $\emptyset :=_{df} 0, \{\emptyset\} :=_{df} 1$, $\{\emptyset, \{\emptyset\}\} :=_{df} 2$, etc. The operation "$'$" corresponds to the operation of addition "+1" [9]. Every natural number n contains all numbers preceding it, $1 = \{0\}$, $2 = \{0, 1\}$, etc. At that point Frege considers numbers as classes.

4. Relational Language

Until now, natural numbers have always been introduced in terms of concepts and relations. Perhaps we can introduce them using only relations. Let us construct now a new language totally based on an abstract concept of relation. We will begin with two primitive notions: quality and binary relation as conceived by De Giorgi et al. [3], [5].

1. We will say that if q is a quality, then $q\,x$ means that the object x has the quality q. If nn is a quality to be a natural number then by writing $nn1$ we state that 1 is a natural number.
2. Given two objects x, y of any nature and a binary relation r, we will write $r\ x, y$ to say that "x and y are in the relation r". Sometimes instead of saying x and y are in the relation r we will say that x is in the relation r with y, for ex. $A \subseteq B$ states that $\subseteq A, B$, i.e. A and B are in a binary relation of inclusion – $\subseteq$.

Following [3], [5] we can introduce fundamental relations: $Rqual, Rrelb$, Rid which describe the behavior of qualities and relations.

Axiom 6

$Rqual$ is a binary relation.

1. if $Rqual\ x, y$ then $Qqual\ x$,
2. if $Qqual\ q$ then $Rqual\ q, x \equiv q\,x$.

Hence, q is a quality and x is a variable and they stand in a binary relation $Rqual$.

Axiom 7

$Rrelb$ is a ternary relation.

1. if $Rrelb\ x, y, z$ then $Qrelb\ x$,
2. if $Qrelb\ r$ then $Rrelb\ r, x, y \equiv r\ x, y$.

Axiom 8

Rid is a binary relation such that: $Rid\ x, y$ holds if and only if x and y are the same object.

$Rrelb$ is defined as a ternary relation which describes the behavior of a binary relation (cf. Axiom 7). On the other hand both $Rqual$ and Rid are originally defined as binary relations (Axioms 6, 8). We can note that the introduction of fundamental relations changes the perspective of considering the entities. In a certain sense it is an abstraction from the entities related in which both entities and the relation between them are considered on the same level. Hence, it would seem natural to consider *qual* and *id* as objects of a type we can call *unary relation* (for more details see [13], [14]) which can be defined as follows:

Definition 2

A unary relation is any "relation" $*$ such that $R*$ is a binary relation.

In the expression $Rrelp\ p, z$ $Rrelp$ indicates the fundamental relation (binary) connecting the unary relation p and its argument z (In [14] we called such types of relations *primary relations* to underlie their role in a system). One can notice, that unary relations underlie those binary relations in which at least one of the objects is defined in terms of the other. In literature [17] one can also find the term *unary relation* applied for subsets of a given set. In this perspective, the identity relation (cf. Axiom 8) will be defined as follows:

Axiom 9

id is a unary relation. Rid is a one-one binary relation for which $Rid\ x, y$ **iff** x and y are the same object.

Defining a concept of unary relation permits us to introduce another specific form of this relation, called tr relation and used for a definition of a kind of dynamic identity [14].

Axiom 10

tr is a unary relation. Rtr is a *one-one* binary relation such that: $Rtr\ x, y$ **implies** that x is not y (x and y are NOT the same object).

Now, we will give very briefly an outline of a definition of dynamic identity which will be used to construct natural numbers.

Definition 3

The dynamic identity triple DIT is composed of three distinct tr relations: tr_1, tr_2, tr_3.

Axiom 11

tr_1 is described by the binary relation Rtr_1 or, alternatively, by the unary operation[1] $Optr_1$ which acts in the following way: If $Rtr_1\ x, y$ then $Optr_1(x) = y_x$ where y_x means "y with x in y", which is to be interpreted in the (*mereological*) sense [10] of x being a part of y.

Axiom 12

tr_2 is described by the binary relation Rtr_2 or, alternatively, by the unary operation $Optr_2$ which acts on the result of $Optr_1$ in the following way: If $Rtr_2\ y, x$ then $Optr_2(y_x) = x_{y_x}; Optr_2$ transforms the result of

[1] We apply the concept of one-to-one correspondence or unary operation as it is conceived in a classical approach (see [17]) and describe in an axiomatic way the behavior of these three types of tr relations. We have adopted the original way of describing it.

$Optr_1(x)$, i.e. y_x, into a singular (mereological) class, (see [14], [10]) and gives as a result x_{y_x}.

Finally,

Axiom 13

Given tr_1 and tr_2, there is another tr relation tr_3 such that $Rtr_3\ z, tr_1$ iff $Rtr_3\ z, tr_2$.

Summarizing, $Optr_1$ acts as a function mapping y into x. $Optr_2$ transforms the result of $Optr_1$ into a class composed of only one element – itself. The image of $Optr_2$ is the generator with what has been generated by it in it. tr_3 is a pure relationship between tr_1 and tr_2, it is not a map. The main idea how these three relations behave is that any change of one's features causes changes in two other entities because of their mutual dynamic relationship. Such a model was thought for physical phenomena, in particular for elementary particles.

5. Natural Numbers in Relational Language

Now we will try to adopt this relational framework to construct natural numbers. We will show that the statements of Peano's system become theories in this new language. This part will be slightly formal.

Let us abbreviate some expressions. Let us put at the place of $Optr_1 - f$, $Optr_2 - \varphi$ and ($Rtr_3\ z, x$ iff $Rtr_3\ z, y$) – $x \Diamond y$

If $tr_1 = x$ then by Axioms 11, 12, 13

1. $f(x) = y_x$ – which means x is a proper part of y.
2. $\varphi(f(x)) = \varphi(y_x) = x_{y_x} = x_{f(x)}$ – which represents a singular (mereological) class in which $f(x)$ is a proper part of x and is "identical" to x in a sense that there is no difference between them [14] in this universe.
3. $y_x \Diamond x_{y_x} \Leftrightarrow f(x) \Diamond x_{f(x)}$. Hence, we can substitute $x_{f(x)}$ by $f(x)$ because of the equivalence relation.

We can construct natural numbers in the following way:

$x :=_{df} 0$
$f(x) = f(0) = y_x :=_{df} 1$
$\varphi(f(x)) = \varphi(f(0)) = \varphi(1) = x_{y_x} = x_{f(x)} = x_1 = 0_1$
$1 \Diamond x_1$

$f(x_1) = f(1) :=_{df} 2$
$\varphi(f(x_1)) = \varphi(f(1)) = \varphi(2) = x_{f(x_1)} = x_2 = 0_2$
$2\Diamond x_2$

$f(x_2) = f(2) :=_{df} 3$
$\varphi(f(x_2)) = \varphi(f(2)) = \varphi(3) = x_{f(x_2)} = x_3 = 0_3$
$3\Diamond x_3$, etc. ...

We obtain the following sequence, beginning with 0, composed of natural numbers (tr_1 relations) and singular classes (tr_2 relations).

$x = 0 \to_f 1 \to_\varphi x_1 \to 2 \to x_2 \to 3 \to x_3...$

We can notice that the natural number n and its corresponding singular class x_n form an equivalence class[2].

We shall introduce now some simple definitions and assumptions in order to define natural numbers.

Axiom 14

0 is a natural number.

Definition 4

A natural number (beginning from 1) is a tr_2 relation.

[2] (1) $\Diamond$ is reflexive: $x\Diamond x$. By Axiom 13: Rtr$_3$ z,x $\Leftrightarrow$ Rtr$_3$ z,x, which is a tautology.

(2) $\Diamond$ is symmetric: $x\Diamond y \Rightarrow y\Diamond x$. Let us put a: Rtr$_3$ z,x $\Leftrightarrow$ Rtr$_3$ z,y, b: Rtr$_3$ z,y $\Leftrightarrow$ Rtr$_3$ z,x. Hence, we have to prove that $a \Rightarrow b$. By Axiom 13 we have even more than a simple implication: $a \Leftrightarrow b$.

(3) $\Diamond$ is transitive: $x\Diamond y$ and $y\Diamond r \Rightarrow x\Diamond r$. Let us put a: Rtr$_3$ z,x $\Leftrightarrow$ Rtr$_3$ z,y, b: Rtr$_3$ z,y $\Leftrightarrow$ Rtr$_3$ z,r, c: Rtr$_3$ z,x $\Leftrightarrow$ Rtr$_3$ z,r. We have to prove that $a \wedge b \Rightarrow c$. We can make the following substitution applying some substitution rules for equivalence, implication and De Morgan Laws [17] (for a moment, let us substitute Rtr_3z, x by zx, Rtr_3z, y by zy and Rtr_3z, r by zr):

$(zx \Leftrightarrow zy) \equiv (zx \Rightarrow zy) \wedge (zy \Rightarrow zx) \equiv\sim (zx\wedge \sim zy)\wedge \sim (zy\wedge \sim zx) \equiv (\sim zx \vee zy) \wedge (\sim zy \vee zx)$

Thus: $(zx \Leftrightarrow zy) \wedge (zy \Leftrightarrow zr) \equiv (\sim zx \vee zy) \wedge (\sim zy \vee zx) \wedge (\sim zy \vee zr) \wedge (\sim zr \vee zy) \equiv (\sim zx \vee zy) \wedge (\sim zy \vee zr) \wedge (\sim zr \vee zy) \wedge (\sim zy \vee zx) \equiv\sim (zx\wedge \sim zy)\wedge \sim (zy\wedge \sim zr)\wedge \sim (zr\wedge \sim zy)\wedge \sim (zy\wedge \sim zx) \equiv (zx \Rightarrow zy) \wedge (zy \Rightarrow zr) \wedge (zr \Rightarrow zy) \wedge (zy \Rightarrow zx) \equiv (zx \Rightarrow zr) \wedge (zr \Rightarrow zx) \equiv (zx \Leftrightarrow zr)$

Definition 5

Let $i\Diamond x_i$, and g be a successor function defined as follows:
$g(0) = f(0) = 1$
$g(1) = f(x_1) = 2$
$g(2) = f(x_2) = 3$
etc. ...
$g(n) = f(x_n) = n + 1$

If we consider P to be a set of natural numbers then we state that:

Corollary 1

$\langle P, 0, g\rangle$ is a model of Peano's axioms.

Proof.

1. The first Peano axiom (Axiom 1) is assured by Axiom 14.
2. For every natural number n, such that $n \geq 1$ if n is a tr_2 relation then by Definition 5: $g(n) = f(x_n)$, which by Axiom 11 is a tr_2 relation. Thus $g(n)$ is also a tr_2 relation.
3. 0, by construction, is a successor of no natural number.
4. Let k, l be natural numbers such that $g(k) = g(l)$.
   Let us notice that $g(k) = f(x_k)$ and $g(l) = f(x_l)$, $g(k) = g(l)$ implies that $f(x_k) = f(x_l)$. Because $x_k\Diamond k$ (x_k can be substituted by k [7]) then $f(x_k)$ becomes $f(k)$. This implies that $f(k) = f(l)$. By Axioms 10, 11, f is '1-1' which implies that $k = l$.
5. Let A be such that: 0 is an element of A and if n is an element of A then $g(n)$ is an element of A. In other words if n is a natural number then $g(n)$ is also a natural number. From general rules for quantifiers [17] we will have: $\forall n$ such that n is a natural number, $g(n)$ is a natural number.
   Summarizing: 0 is an element of A and for every n if n is a natural number then $g(n)$ is also a natural number (cf. (2)), so every natural number is an element of A. □

6. Discussion of Results

Coming back to Frege, **the number** of the concept F is defined as the extension or **set of all concepts that are equinumerous with F**. This number **is also identified with the class** of all concepts under which **n** objects fall. In this way the number 0 is identified with the number of the concept of not being self-identical, the number 1 is identified with the class of all concepts for which exactly one thing falls under F, etc.

We have defined tr_1 as x: $tr_1 = x :=_{df} 0$, $tr_2 = f(x) = y_x :=_{df} 1$. In this sense 1 could be interpreted as a one-to-one correspondence between x and y. We can do that because of mapping between x and y. Moreover, we have showed that $\varphi(y_x) = x_{y_x} = x_1$, is a singular (mereological) class, a class composed only of its unique element – itself. Finally, $y_x \Diamond x_{y_x} \Leftrightarrow 1 \Diamond x_1$ – the number 1 is identified with this singular class x_1, etc. In this perspective natural numbers reflect entities describing a relationship between more basic concepts: *tr* relations. In this way the number 1 is not interpreted as a number of elements, but as a set of all concepts for which we have one-to-one correspondences. This interpretation is nothing else than Frege's concept of number. Hence, we have constructed natural numbers as conceived by Frege, but in a totally different way. How do we approach these results?

A great number of authors would base their evaluation on the correctness of the way of reasoning. Sometimes reasoning enables to produce arguments to convince others and to accept valuable information, but sometimes we point out a problem not to convince others of the truth of our opinion, but to meet the challenges of others [12]; the opinion of a group is always better than even that of its best member.

Hence, we can pose a problem in another way. In the example described above we have used arguments that are true, we followed the rules of classical logic. On the other hand the results might seem not to be strong enough to convince about the validity of the presented theory although our reasoning was based on general knowledge and we made decisions based on them. We have defined a language and rules, and we followed them. At this point we can behave like reviewers of scientific manuscripts, we can look for flaws in different papers either to justify or to reject results. The problem is that perhaps there is little empirical research on this topic or there is poor performance – the lack of elegance and beauty required in mathematical papers. By the way, the last one plays a great role in decision making [8].

The discussed example, hence, is an exemplification of a case where there is little empirical research on the topic. The relational language is new, very abstract, not well known enough, and this might also explain some lack of conviction in accepting results. There are so many different theories. How to gain the certaintity that we are not wrong? When the same problems are placed in a well known argumentative setting, a theory turns out to be appropriate and innovative which is not the case in a not well known context.

In the latter it seems that even the truth cannot support a theory. Even exact and formal argumentation cannot defend the opinions. Perhaps this explains some resistance of the scientific society in the face of new inventions, even if the motivation is pure curiosity about what is true. Nowadays, more often than in the past, authors are asked to give applications of their theories and this takes something away from the beauty of doing research and discovering the world. Like Johnson, I have to admit that scientists, which means also me, are interested in truths and when they discover something they would like to convince others of that truth. In general, “pure” mathematicians are not interested in applications of their theories but are attracted by the logic of the world and the beauty of discovery.

It happens that the only argument we can use to convince others of our results is time, which is not logical. Time will show whether results are worth to accept or not, as has happened many times in our history. However, argumentations motivated in this way can distort evaluations and attitudes and allow erroneous beliefs to persist. They are not in favour of approving of some decisions.

It might not be expected that even in science, argumentations consisting of truths sometimes are not strong enough to favour the conclusions for which they were found. Perhaps everything was calculated in our brain [16]. Perhaps the united forces of logicians, scientists, physicists and philosophers, one day, will transgress this limitation [11]. Anyway, it is comfortable to know that at the end truth should win, and this gives great satisfaction and courage to scientists to continue doing research.

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KNOWLEDGE AND INTUITIONISTIC TENSE LOGIC

Abstract. In this paper we describe the system of intuitionistic tense logic and consider the possibility of using this system to represent the changing of knowledge over time.

Keywords: knowledge, tense logic, intuitionistic logic, epistemic logic.

Most of the systems used to formalize knowledge changing in time are constructed over classical propositional logic. It seems natural to ask whether this is the only possible way to construct logical systems for this task. We will show that this task can be accomplished even when as a basis we use intuitionistic propositional logic instead of classical propositional logic.

We will describe the intuitionistic tense logic system and demonstrate that this system can be used for formal description of knowledge changing over time, although in this system there are no epistemic operators. The representation of knowledge in our system is not realized at a syntactic level, but due to the properties of intuitionistic logic, knowledge is represented at the semantic level. This approach is the result of a proposed semantics for intuitionistic logic in which are used notions like *proof*[1], *information*, and *knowledge*.

For the considered system of intuitionistic temporal logic, Kripke-style semantics is proposed. This type of semantics is also the proper semantics for intuitionistic propositional logic. Kripke models [3] for intuitionistic propositional logic are similar to Kripke models for modal logic constructed over classical propositional logic. In these models we have a collection of the worlds W and the relation of accessibility R while, in the case of intuitionistic logic, the elements of the set W we consider rather as states of information, knowledge, etc, than as possible worlds. The R relation between elements w and v (wRv) is interpreted as *w has access to v*, which

[1] The semantics proposed by Kolmogorov.

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means that the information state v is an available extension of the information state w. The crucial difference between Kripke's models for intuitionistic logic and Kripke models for modal logic constructed over classical propositional logic is in the fact that in the case of modal logic constructed over classical propositional logic the relation R is used only to interpret the modal operators; for intuitionistic logic, this relation is used to interpret intuitionistic connectives: negation and implication.

A formula $\neg\varphi$ is true[2] in a certain information state w if there is no information state available from the state w such that φ is true at this state. In other words, the formula $\neg\varphi$ is true in the state w if there is no possibility that φ is true in any information state available from the state w.

We have the same in the case of intuitionistic implication. The formula $\varphi \to \psi$ is true in the information state w when in any information state available from w, the truth of φ implies the truth of ψ. Moreover, in Kripke models for intuitionistic logic is built a condition of monotonicity. The true (forced) formula in a given information state remains truthfulness in any extension of this state.

Modality in intuitionistic logic, we can see, for example, in the syntactic definition of intuitionistic negation. The formula $\neg\varphi$ is syntactically equivalent to the formula $\varphi \to \bot$. Thus, the intuitionistic negation can be seen as a kind of impossibility operator.

Kripke-style semantics for intuitionistic propositional logic is as follows:

1. Kripke semantics for intuitionistic propositional logic

The intuitionistic model is a triple $\mathfrak{M} = \langle W, \leq, V \rangle$. The truth of the formula φ in a model $\mathfrak{M}$, in a state s we define as follows:

$$\mathfrak{M}, s \models \quad \equiv \quad s \in V(p), \text{ where } p \text{ is a propositional letter,}$$

$$\mathfrak{M}, s \models \neg\varphi \quad \equiv \quad \underset{s \leq s'}{\forall} \mathfrak{M}, s' \not\models \varphi,$$

$$\mathfrak{M}, s \models \varphi \wedge \psi \quad \equiv \quad \mathfrak{M}, s \models \varphi \text{ and } \mathfrak{M}, s \models \psi,$$

$$\mathfrak{M}, s \models \varphi \vee \psi \quad \equiv \quad \mathfrak{M}, s \models \varphi \text{ or } \mathfrak{M}, s \models \psi,$$

$$\mathfrak{M}, s \models \varphi \to \psi \quad \equiv \quad \underset{s \leq s'}{\forall} (\text{if } \mathfrak{M}, s' \models \varphi, \text{ then } \mathfrak{M}, s' \models \psi).$$

Let us note that in intuitionistic logic, if a formula $\neg\varphi$ is true at some information state, then we know not only that in the current information

[2] Intuitionistic logic also uses the term *forced*.

state φ is not satisfied (this information we obtain in the case of classical logic), but furthermore we know that the formula φ will never be satisfied. Our *never* refers to all available extensions of the current information state. Aside from information given explicitly in intuitionistic logic, we have additional information. This feature of intuitionistic logic van Benthem called *implicit knowledge* [1]. For expression of this kind of knowledge in the language of intuitionistic logic, we do not need any additional specific operators. Although similar semantics, this feature strongly differentiates intuitionistic logic from epistemic logic based on classical logic. In the language of epistemic logic we can express *explicit knowledge*, and we use for it the epistemic operator K. The language of intuitionistic logic allows us to express interesting notions without referring to epistemic operators. For example from the truth of the formula $\neg\neg\varphi$ we can conclude that for each state there is such an extension, in which φ is true. This statement is, omitting details, close to *we know that* φ *must be true.*

In Kripke's semantics for epistemic logic based on classical propositional logic, the satisfiability of the formula $K\varphi$ in model $\mathfrak{M}$, in the information state s we define as follows:

$$\mathfrak{M}, s \models K\varphi \qquad \equiv \qquad \underset{s \leq s'}{\forall} \mathfrak{M}, s' \models \varphi.$$

Let us consider the truth of the formula $K\neg\varphi$ in the model $\mathfrak{M}$, in the state s. According to the definition of satisfiability of K operator we have:

$$\mathfrak{M}, s \models K\neg\varphi \qquad \equiv \qquad \underset{s \leq s'}{\forall} \mathfrak{M}, s' \models \neg\varphi.$$

If we regard the definition of satisfiability for negation in epistemic logic we have:

$$\mathfrak{M}, s \models K\neg\varphi \qquad \equiv \qquad \underset{s \leq s'}{\forall} \mathfrak{M}, s' \not\models \varphi.$$

From the definition of satisfiability of negation in intuitionistic logic

$$\mathfrak{M}, s \models \neg\varphi \qquad \equiv \qquad \underset{s \leq s'}{\forall} \mathfrak{M}, s' \not\models \varphi.$$

we see that the intuitionistic negation ($\neg$) may, in some sense, be regarded as a combination of the K operator and classical negation ($K\neg$). We can make a similar reasoning for intuitionistic implication and show that the intuitionistic formula $\varphi \rightarrow \psi$ can be, ignoring the details, regarded as a *modalized implication* or a combination of epistemic operator K and classical implications: $K(\varphi \rightarrow \psi)$.

2. IK'_t - intuitionistic tense logic

Now we describe the system of intuitionistic tense logic. This system is a simpler version of Ewald's system [2], described in [4], [5], [6].

2.1. Syntax

An alphabet $\mathcal{A}$ of the language of $\mathfrak{L}_{IK'_t}$:

- a countable set of propositional letters $\mathcal{AP}$,
- unary connective: $\neg$,
- binary connectives: $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$,
- tense operators: G, H, F, P,
- parentheses: $), ($.

Definition 2.1

The set $FOR(\mathfrak{L}_{IK'_t})$ is the smallest set of finite sequences of elements of the alphabet $\mathcal{A}$ such that:

- $\mathcal{AP} \subseteq FOR(\mathfrak{L}_{IK'_t})$,
- if $\varphi, \psi \in FOR(\mathfrak{L}_{IK'_t})$, then $\neg\varphi, G\varphi, F\varphi, H\varphi, P\varphi, \varphi \wedge \psi, \varphi \vee \psi, \varphi \rightarrow \psi, \varphi \leftrightarrow \psi \in FOR(\mathfrak{L}_{IK'_t})$.

Tense operators are interpreted as usual:

$G\varphi$ – always in the future φ,

$F\varphi$ – sometime in the future φ,

$H\varphi$ – always in the past φ,

$P\varphi$ – sometime in the past φ.

2.2. Semantics

We use IK'_t system to describe changing states of knowledge. Changing knowledge in IK'_t is understood as a transition to the next state of knowledge. It is assumed that whole knowledge of the current state of knowledge is available in every state of knowledge which is no less than the current state. We assume monotonicity of the process of acquiring knowledge. A bigger state of knowledge we reach by enriching knowledge of an agent with new facts. This may occur in some cases.

We can enrich the knowledge through research when we make a description of past events which took place at time points that were not known in the state of knowledge. In the given state of knowledge we had no information about these events. In this case, the structure of time of the new state of knowledge expands in the past and is a superset of the time structure of the new state.

We can know new elements of the future time. In this case, the structure of time of a new state of knowledge expands in the future, and the relation of inclusion of time structure is similar to the first case.

We can learn new moments of time from the past and from the future. In this case, the time structure of the not lesser state of knowledge expands, both in the past and in the future.

In each case under consideration the increase in the level of knowledge induces appropriate inclusions of temporary structures. The expansion of the structure of time (regardless of the direction in which it occurs) changes the relation of temporal succession. In the new state of knowledge we therefore have to consider the changed relation.

Another possible option to achieve a new state of knowledge is a case when the set of moments of time is not changing, but the power of sets of events mapped to moments of time is increasing.

The state of knowledge in the proposed semantics is conceived as consisting of a set of facts, which are semantic correlates of sentences, a collection of moments of time and the relation of temporal succession. A subset of a particular set of facts mapped to a moment of time is conceived as a set of facts known at this moment.

We want to construct a language which we use to describe the states of knowledge changing over time. Assume therefore, that the state of knowledge, let's call it m, is not a total state of knowledge, so there are unknown facts in this state. This implies the possibility of multiple states of knowledge, different from m. In these states of knowledge the agent knows all facts which are known in the state m; additionally, there are also known new facts. The main possible differences between initial state m and new states of knowledge are: a bigger set of known facts, a set of new moments of time, and a set of new moments of time which are in the relation *before-after*. Such states of knowledge we call states reachable from the state m. We also accept that the state m is reached from m. This means that the relationship is reflexive. If this assumption is not accepted, we reject the possibility the state m is not total. We do not want, however, logic IK_t' to resolve it. The logic is to be independent in this respect.

In the states of knowledge reachable from m there are no less known facts than in the state m. We say that these states of knowledge are not lesser than m, or that they are states of knowledge in which the level of knowledge is not lesser than the level of knowledge in the state m.

A state of m', reachable from a state m may also not be a total state of knowledge. From the point of view of the state of m' reachable can be state m'' such that in m'' is known everything that it is known in the state of m' and also new events are known which are not known in the state m'. Since everything that is known in the state m is also known in the state m' and everything that is known in the state m' is known in the state m'', therefore everything that is known in the state of m is also known in the state m''. State m'' is therefore reachable from state m. So we conclude that the relation*before-after* is transitive. Since the relation *before-after* is reflexive and transitive, then it is a part-ordering relation.

We say that the state m'' has a not lesser level of knowledge than the state m', if m'' meets the following conditions:

1. A set of moments of time of state m' is included in the set of moments of time of the state m''. (Changing the number of moments of time changes the state of knowledge.)
2. In the state of m'' are preserved - occurring between time moments - connections *before-after*, which existed in the state m'. Moreover, in the state m'' may occur before-after connections, which did not hold in the state m'.
3. All events which are known in the state m' are known in the state m''. (Everything that is known does not cease to be known, even if there are new events known.) Moreover, in the moments of time of state m'' may be known events, which are not known in the equivalents of those moments in the state m'.

Between the conditions 1), 2) and 3) there are certain connections.

The fulfillment of condition 1) implies condition 2), because in our discussion we omit cases where *new* moments of time are not connected in the relation *before-after* with other moments of time. The change of the set of moments of time involves a change in the relation before-after. However, the change of the relation of temporal succession does not involve change of the set of moments of time. In a state of knowledge with no lesser level of knowledge, new connections may occur between moments of time existing in the state of knowledge with a lesser level of knowledge. Condition 2) does not therefore fulfill condition 1). Similarly, condition 3) does not imply condition 1) or 2), as new events can be known without the existence of new moments of time or new connections *before-after*.

Each moment of time is assigned to a non-empty set of known events. If there are new moments of time, there are also new events known. Condition 1) implies therefore condition 3).

The existence of new connections *before-after* implies the existence of new known events in those moments of time in which new connections take place. Thus, condition 2) implies condition 3).

In our system we have two kinds of time. First, is the time that is assigned to a state of knowledge. This is a structure consisting of a collection of moments which are in the relation of temporal succession of a given state of knowledge. Second, is the time that is not relativized to any state of knowledge. This time is the sum of the time assigned to all possible states of knowledge.

These intuitions we describe in a formal way.

- I is a nonempty set.
- T_i $(i \in I)$ is a nonempty set.
- $R_i (\subseteq T_i \times T_i)$.
- $\mathcal{T}_i = \langle T_i, R_i \rangle$.
- $T = \bigcup_{i \in I} T_i$.
- $R = \bigcup_{i \in I} R_i$.
- $\mathcal{T} = \langle T, R \rangle$.
- $V_i \subseteq T_i \times 2^{\mathcal{AP}}$, where $i \in I$.
- $\mathcal{F} = \{V_i : i \in I\}$.
- $m_i = \langle T_i, R_i, V_i \rangle$ where $i \in I$.
- $\mathfrak{M} = \{\langle T_i, R_i, V_i \rangle : V_i \in \mathcal{F}, i \in I\}$, so $\mathfrak{M} = \{m_i : i \in I\}$.

I is a set of indexes of states of knowledge. T_i is the set of moments of time in the state of knowledge indexed by i. R_i is a binary relation on the set of moments of time in a state of knowledge indexed by i. The relation R_i is understood as a *before-after* relation on the set of moments of time of the state of knowledge indexed by i. $\mathcal{T}$ is the time in the state of knowledge indexed by i. T is the set of all moments of time of all states of knowledge. R is a binary relation on T. Let us remark, that $R \subseteq T \times T$. $\mathcal{T}$ is the sum of times of all states of knowledge. V_i is a function mapping to moments $t \in T_i$ subsets $V_i(t)$ of the set of propositional letters. $\mathcal{F}$ is a set of valuations. m_i is the state of knowledge indexed by i. $\mathfrak{M}$ is a model based on time $\mathcal{T}$ and a class of function $\mathcal{F}$.

In model $\mathfrak{M}$ we define relation $\leq (\subseteq \mathfrak{M} \times \mathfrak{M})$

Definition 2.2

For any $i, j \in I$:

$$m_i \leq m_j \equiv (T_i \subseteq T_j, R_i \subseteq R_j, \forall_{t \in T_i} V_i(t) \subseteq V_j(t)).$$

The fact that the states m_i, m_j are in the relation $\leq$ ($m_i \leq m_j$) is understood as follows: the state m_j has no lesser level of knowledge than the state of m_i.

The relation $\leq$ is determined by the inclusions: a set of moments of time, relations of temporal successions and sets of events known at particular moments in time. So, the relation $\leq$ has all the properties of inclusions. In particular $\leq$ is reflexive and transitive.

Theorem 1

For any $m_i (\in \mathfrak{M})$:

$$m_i \leq m_i.$$

Theorem 2

For any $m_i, m_j, m_k (\in \mathfrak{M})$:

$$\text{if } (m_i \leq m_j \text{ and } m_j \leq m_k), \text{ then } m_i \leq m_k.$$

The relation $\leq$ partially orders the set of states of knowledge. Let us consider some possible cases when $m_i \leq m_j$.

The first possible case is:

$$T_i = T_j \text{ and } R_i = R_j \text{ and } \forall_{t \in T_i} (V_i(t) \subseteq V_j(t)).$$

This occurs when the sets of moments of time of states of knowledge m_i and m_j are the same ($T_i = T_j$). The same in both states of knowledge is the relation ($R_i = R_j$). State m_j is formed by changing the value of the V_i. In other words, in this case, the state of knowledge m_j is obtained by increasing the number of known facts at particular moments in time.

Another possible situation is the following:

$$T_i \subseteq T_j, R_i \subseteq R_j \text{ and } \forall_{t \in T_i} (V_i(t) \subseteq V_j(t)).$$

In this case, the state m_j is formed by adding new moments of time to the time structure of the state m_i. At no moment of time $t\,(\in T_i)$ does not change the set $V_i\,(t)$. Change of a level of knowledge is based on the occurrences in the state m_j of new moments of time (in the future or in the past). Because in the state of knowledge m_j we have new moments of time, all components of m_i change. We have a change in the set of moments of time. We have a change in the relation *before-after*, because some moments of time of the state m_i will be in relation *before-after* with new moments of time. And finally we have a change in the V_i function because its domain is changed (subsets of a set of propositional letters will be mapped to new moments of time).

Yet another possibility is:

$$T_i = T_j, R_i \subseteq R_j \text{ and } \underset{t \in T_i}{\forall} (V_i(t) \subseteq V_j(t)).$$

It might also be that the reason for change of a level of knowledge of a state of knowledge m_i is not a change of the set of moments of time of m_i but a change of the relation *before-after*. The change of the relation *before-after* involves known facts in these moments of time which have new connections with other moments of time.

Abbreviation

By m_i^* (where $i \in I$) we means any m_j ($\in \mathfrak{M}$) such that

$$m_i \leq m_j.$$

We now give a definition of the satisfiability of a sentence in the model[3].

Definition 2.3

Satisfiability of the sentence φ in the model $\mathfrak{M}$, in the state m_i, in the moment t we define in the following way:

a) $\mathfrak{M}, m_i, t \models \varphi \quad \equiv \quad \varphi \in V_i(t)$, if $\varphi \in \mathcal{AP}$,

b) $\mathfrak{M}, m_i, t \models \neg\varphi \quad \equiv \quad \underset{m_i^* \in \mathfrak{M}}{\forall} \mathfrak{M}, m_i^*, t \not\models \varphi$

c) $\mathfrak{M}, m_i, t \models \varphi \vee \psi \quad \equiv \quad \mathfrak{M}, m_i, t \models \varphi$ or $\mathfrak{M}, m_i, t \models \psi$,

d) $\mathfrak{M}, m_i, t \models \varphi \wedge \psi \quad \equiv \quad \mathfrak{M}, m_i, t \models \varphi$ and $\mathfrak{M}, m_i, t \models \psi$,

e) $\mathfrak{M}, m_i, t \models \varphi \to \psi \quad \equiv \quad \underset{m_i^* \in \mathfrak{M}}{\forall} (\mathfrak{M}, m_i^*, t \not\models \varphi \text{ or } \mathfrak{M}, m_i^*, t \models \psi)$,

f) $\mathfrak{M}, m_i, t \models F\varphi \quad \equiv \quad \underset{t_1 \in T_i}{\exists} (tR_it_1 \text{ and } \mathfrak{M}, m_i, t_1 \models \varphi)$,

g) $\mathfrak{M}, m_i, t \models G\varphi \quad \equiv \quad \underset{m_i^* \in \mathfrak{M}}{\forall} \underset{t_1 \in T_i^*}{\forall} (\text{if } tR_i^*t_1, \text{ then } \mathfrak{M}, m_i^*, t_1 \models \varphi)$,

h) $\mathfrak{M}, m_i, t \models P\varphi \quad \equiv \quad \underset{t_1 \in T_i}{\exists} (t_1R_it \text{ and } \mathfrak{M}, m_i, t_1 \models \varphi)$,

i) $\mathfrak{M}, m_i, t \models H\varphi \quad \equiv \quad \underset{m_i^* \in \mathfrak{M}}{\forall} \underset{t_1 \in T_i^*}{\forall} (\text{if } t_1R_i^*t \text{ then } \mathfrak{M}, m_i^*, t_1 \models \varphi)$.

We now give some basic definitions.

[3] This definition uses quantifier symbols: $\forall$ – *for any*, $\exists$ – *exists*. The symbols $\forall$ and $\exists$ are not symbols of the language of IK_t' system. We use them as metalanguage symbols. Moreover, we use the symbol $\not\models$. By $\mathfrak{M}, m_i^*t, \not\models \varphi$ we mean *it is not true, that* $\mathfrak{M}, m_i^*, t \models \varphi$.

Definition 2.4

$\mathfrak{M} \models \varphi$ (φ is true in a model $\mathfrak{M}$) iff for any state of knowledge $m_i (\in \mathfrak{M})$ and for any $t (\in T_i)$ holds $\mathfrak{M}, m_i, t \models \varphi$.

Definition 2.5

$\mathcal{T} \models \varphi$ (φ is true in time $\mathcal{T}$) iff φ is true in a model $\mathfrak{M}$ for any nonempty class of function $\mathcal{F}$ $(= \{V_i : i \in I\})$.

Definition 2.6

$\models \varphi$ (φ is true) iff for any $\mathcal{T}$, $\mathcal{T} \models \varphi$.

Between tense logic systems based on classical logic and tense logic systems based on intuitionistic logic, there are many differences. One of them is that in the intuitionistic tense logics falsehood $\neg\varphi$ does not imply the truth of φ.

Let us consider some particular case. A sentence φ is not known in the state m_i at the time $t (\in T_i)$, however φ is known at this time in the state m_j which has no lesser level of knowledge than the level of knowledge of the state m_i. If from the fact that φ is not known at the time t in the state of knowledge m_i we conclude that in this state of knowledge at time t $\neg\varphi$ is known, then – according to the definition of satisfiability – φ could not be known at t in any state of knowledge m_i^*. In particular, the sentence φ could not be known at the time t in the state m_j. This leads to a contradiction, because we get the sentence φ is known and unknown in t in the state m_j. If φ is known at some moment of time in state m_i, then in each state of knowledge m_i^* at t the sentence φ is known. However, if at some moment of time the sentence φ is not known, then it does not mean that at this moment, in every state of knowledge m_i^* the sentence φ is known.

Now we prove the lemma, which expresses the monotonicity of knowledge in the system IK_t'. What is known in the state m_i is also known in every state of knowledge with a level of knowledge which is not lesser than the level of knowledge of the state m_i.

Lemma 1

For any φ and for any $m_i, m_j \in \mathfrak{M}$:

$$\text{if } (m_i \leq m_j \text{ and } \mathfrak{M}, m_i, t \models \varphi), \text{ then } \mathfrak{M}, m_j, t \models \varphi.$$

Proof.

We prove this by induction with respect to the length of φ. Let us assume that $m_i \leq m_j$.

($\varphi \in \mathcal{AP}$)

First, let us consider a case when φ is a propositional letter.
From the definition 2.2 if $m_i \leq m_j$, then for any $t \in T_i$ holds

$$V_i(t) \subseteq V_j(t). \tag{1}$$

If $\mathfrak{M}, m_i, t \models \varphi$, then from the definition 2.3

$$\varphi \in V_i(t). \tag{2}$$

From (1) and (2) we have

$$\varphi \in V_j(t). \tag{3}$$

Because φ is a propositional letter, then from (3) and the definition 2.3 we have $\mathfrak{M}, m_j, t \models \varphi$.

Inductive assumption:

Let φ, ψ satisfy the following conditions:

a) if $\mathfrak{M}, m_i t \models \varphi$, then $\mathfrak{M}, m_j, t \models \varphi$,

and

b) if $\mathfrak{M}, m_i, t \models \psi$, then $\mathfrak{M}, m_j, t \models \psi$.

Now, let us consider sentences built from φ, ψ, connectives and tense operators.

($\neg\varphi$)

Let us assume that $\mathfrak{M}, m_i, t \models \neg\varphi$.
From the definition 2.3.b we have:

$$\text{for any } m_k, \text{ such that } m_i \leq m_k \text{ it is true, that } \mathfrak{M}, m_k, t \not\models \varphi. \tag{1}$$

Let us consider any state of knowledge m_l with a level of knowledge not lesser than the level of knowledge of the state m_j,

$$m_j \leq m_l. \tag{2}$$

From (2), the assumption $m_i \leq m_j$ and a transitivity of relation $\leq$ we have $m_i \leq m_l$. Thus, from (1) we have $\mathfrak{M}, m_l, t \not\models \varphi$. Because m_l is any state of knowledge with a level of knowledge not lesser than the level of knowledge of m_j, we have:

$$\text{for any } m_l \text{ such that } m_j \leq m_l \text{ is true, that } \mathfrak{M}, m_l, t \not\models \varphi. \tag{3}$$

From (3) and the definition 2.3.b we have

$$\mathfrak{M}, m_j, t \models \neg\varphi.$$

$(\varphi \wedge \psi)$

Let us assume $\mathfrak{M}, m_i, t \models \varphi \wedge \psi$.
Thus, from the definition 2.3.d:

$$\mathfrak{M}, m_i, t \models \varphi, \tag{1}$$

and

$$\mathfrak{M}, m_i t \models \psi. \tag{2}$$

From 1) and point a) of inductive assumption we have:

$$\mathfrak{M}, m_j, t \models \varphi. \tag{3}$$

Analogous, from 2) and point b) of inductive assumption we have:

$$\mathfrak{M}, m_j, t \models \psi. \tag{4}$$

From (3), (4) and the definition 2.3.d we have $\mathfrak{M}, m_j, t \models \varphi \wedge \psi$.

$(\varphi \vee \psi)$

Proof is analogous to the case of conjunction.

$(\varphi \rightarrow \psi)$

Let us assume $\mathfrak{M}, m_i, t \models \varphi \rightarrow \psi$.
From the definition 2.3.e we have:

$$\underset{m_i^* \in \mathfrak{M}}{\forall} (\mathfrak{M}, m_i^*, t \not\models \varphi \text{ or } \mathfrak{M}, m_i^*, t \models \psi), \tag{1}$$

Let us consider any state of knowledge m_l with a level of knowledge which is not lesser than the level of knowledge of the state m_j:

$$m_j \leq m_l. \tag{2}$$

From (2), the assumption $m_i \leq m_j$ and the transitivity of the relation $\leq$ we have $m_i \leq m_l$,. Thus from (1) we have $\mathfrak{M}, m_l, t \not\models \varphi$ or $\mathfrak{M}, m_l, t \models \psi$. Because m_l is any state of knowledge with the level of knowledge which is not lesser than the level of knowledge of the state m_j, we have:

$$\text{for any } m_l \text{ such that } m_j \leq m_l \text{ holds: } \mathfrak{M}, m_l, t \not\models \varphi \text{ or } \mathfrak{M}, m_l, t \models \psi). \tag{3}$$

From(3) and the definition 2.3.e) we have $\mathfrak{M}, m_j, t \models \varphi \rightarrow \psi$.

$(G\varphi)$

Let us assume $\mathfrak{M}, m_i, t \models G\varphi$. From the definition 2.3.g:

$$\underset{m_i^* \in \mathfrak{M}}{\forall} \; \underset{t_1 \in T_i^*}{\forall} (\text{ if } tR_i^* t_1, \text{ then } \mathfrak{M}, m_i^*, t_1 \models \varphi), \tag{1}$$

Let us consider any state of knowledge m_l with a level of knowledge which is not lesser than the level of knowledge of the state m_j,:

$$m_j \leq m_l. \tag{2}$$

From (2), assumption $m_i \leq m_j$ and the transitivity of the relation $\leq$ we have $m_i \leq m_l$. Thus, from (1) we have:

$$\text{for any } t_1 (\in T_l) \text{ such that } tR_l t_1 \text{ holds } \mathfrak{M}, m_l, t \models \varphi. \tag{3}$$

Because the state of knowledge m_l is a state of knowledge with a level of knowledge which is not lesser than the level of knowledge of the state of knowledge m_j, we have

$$\underset{m_l}{\forall} \underset{t_1 \in T_l}{\forall} (\text{ if } m_j \leq m_l \text{ and } tR_l t_1, \text{ then } \mathfrak{M}, m_l, t_1 \models \varphi). \tag{4}$$

From (4) and definition 2.3.g we have $\mathfrak{M}, m_j, t \models G\varphi$

$(H\varphi)$

Proof is analogous to the case of the G operator.

$(F\varphi)$

Let us assume $\mathfrak{M}, m_i, t \models F\varphi$. From definition 2.3.f there is a moment $t_1 (\in T_i)$, $tR_i t_1$, such that:

$$\mathfrak{M}, m_i, t_1 \models \varphi. \tag{1}$$

From (1) and point a) of the inductive assumption

$$\mathfrak{M}, m_j, t_1 \models \varphi. \tag{2}$$

From the assumption $m_i \leq m_j$, the definition 2.2 and the definition of inclusion we have:

$$t \in T_j, t_1 \in T_j, tR_j t_1. \tag{3}$$

Thus from (2),(3) and the definition 2.3.f we obtain $\mathfrak{M}, m_j, t \models F\varphi$.

$(P\varphi)$

Proves analogous to the case of the F operator. □

We have shown thus monotonicity of knowledge is described using the language of $\mathfrak{L}_{IK_t'}$. Everything that is known in the state of m_i, is also known in every state of knowledge with a level of knowledge not lesser than the level of knowledge of the state m_i.

2.3. Axiomatization of IK_t'

The IK_t' is axiomatizable. The set of axioms of IK_t' consists of axioms $A1-A10$, which are substitutions of intuitionistic propositional logic axioms and specific axioms $H1-G7$. Rules of inferences in IK_t' are: Modus Ponens MP and temporal generalization rules RH, RG.

2.3.1. Axioms

For any $\varphi, \psi, \gamma \in FOR(\mathfrak{L}_{IK'_t})$:

A1) $\varphi \to (\psi \to \varphi)$,

A2) $(\varphi \to \psi) \to \{[\varphi \to (\psi \to \gamma)] \to (\varphi \to \gamma)\}$,

A3) $[(\varphi \to \gamma) \wedge (\psi \to \gamma)] \to [(\varphi \vee \psi) \to \gamma]$,

A4) $(\varphi \wedge \psi) \to \varphi$,

A5) $(\varphi \wedge \psi) \to \psi$,

A6) $\varphi \to [\psi \to (\varphi \wedge \psi)]$,

A7) $\varphi \to (\varphi \vee \psi)$,

A8) $\psi \to (\varphi \vee \psi)$,

A9) $(\varphi \wedge \neg\varphi) \to \psi$,

A10) $(\varphi \to \neg\varphi) \to \neg\varphi$,

H1) $H(\varphi \to \psi) \to (H\varphi \to H\psi)$,

H2) $H(\varphi \to \psi) \to (P\varphi \to P\psi)$,

H3) $\varphi \to HF\varphi$,

H4) $PG\varphi \to \varphi$,

H5) $P(\varphi \vee \psi) \to (P\varphi \vee P\psi)$,

H6) $(P\varphi \to H\psi) \to H(\varphi \to \psi)$,

H7) $P\varphi \to \neg H\neg\varphi$,

G1) $G(\varphi \to \psi) \to (G\varphi \to G\psi)$,

G2) $G(\varphi \to \psi) \to (F\varphi \to F\psi)$,

G3) $\varphi \to GP\varphi$,

G4) $FH\varphi \to \varphi$,

G5) $F(\varphi \vee \psi) \to (F\varphi \vee F\psi)$,

G6) $(F\varphi \to G\psi) \to G(\varphi \to \psi)$,

G7) $F\varphi \to \neg G\neg\varphi$.

Rules:

Modus Ponens MP: $\dfrac{\varphi \to \psi, \varphi}{\psi}$.

Temporal generalization rules:

$$RH: \quad \frac{\vdash_{IK'_t} \varphi}{\vdash_{IK'_t} H\varphi} \qquad\qquad RG: \quad \frac{\vdash_{IK'_t} \varphi}{\vdash_{IK'_t} G\varphi}.$$

In the K_t system (minimal system of tense logic based on classical propositional logic) specific axioms are $H1$, $G1$, $H3$, $G3$. Other axioms of the IK'_t are theorems of K_t system. Because when we prove these theorems we use some theses of classical propositional logic which are not provable in intuitionistic logic, but are true in any model, then we add these formulas to the set of axioms of IK'_t system. Thus the axioms $H2$, $H4$, $H5$, $H6$, $H7$ and $G2$, $G4$, $G5$, $G6$, $G7$ are also axioms of IK'_t.

The system IK'_t is complete.

Theorem 3 ([5])

Let Σ be a set of sentences of the language $\mathfrak{L}_{IK'_t}$. For any $\varphi \in \Sigma$:

$$\Sigma \vdash_{IK'_t} \varphi \text{ iff } \Sigma \models_{IK'_t} \varphi.$$

3. Conclusion

Intuitionistic logic and knowledge are closely related. An epistemic approach is the epicenter of the intuitionistic Brouwerian explanation of the truth as provable by ideal mathematics, or more generally, the ideal cognitive agent. Intuitionistic Kripke models well model the evolutionary process of acquiring knowledge (information) by agents. It could be asked whether such an approach, the notion of *being true* and *to be known* are to be understood by the ideal cognitive agent as two different terms, or should the terms be equated with each other? It seems that if we use intuitionistic logic for modeling mathematical knowledge only, these two notions are not significantly different. However, when we apply this logic to modeling empirical reasoning, the distinction between these concepts is necessary.

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MANAGING A GRAPH STORE IN THE REST ARCHITECTURAL STYLE

Abstract. This paper describes a simple method of communication between graph store and client over a web. We propose a mechanism based on the Hypertext Transfer Protocol standard. It can be used to express, select, and update operations across various Resource Description Framework (RDF) data sources. We present a fast method that suffices clients to know only Uniform Resource Locator based on Representational State Transfer (REST).

Keywords: Semantic Web, Resource Description Framework (RDF), Representational State Transfer (REST), Graph store, Query languages

1. Introduction

A graph store is a purpose-built database for the storage and retrieval of Resource Description Framework (RDF) data. Much like in the case of other databases, one can find and modify data in graph store via web services.

Following [1], let I be the set of all Internationalized Resource Identifier (IRI) references, B an infinite set of blank nodes, L the set RDF plain literals, and D the set of all RDF typed literals. I, B, L and D are pairwise disjoint. Let $O = I \cup B \cup L \cup D$ and $S = I \cup B$. An RDF triple T is a triple in $S \times I \times O$. If $T = (s, p, o)$ is RDF triple, s is called the subject, p the predicate and o the object. An RDF graph G is a set of RDF triples T. It is collection, which is represented by a labeled, directed multigraph. An RDF graph store GS is a set $\{G_0, (u_1, G_1), (u_2, G_2), \ldots, (u_n, G_n)\}$, where each G_i is a RDF graph and u_i is an IRI reference. Each IRI is distinct. G_0 is called a default graph and each pair (u_i, G_i) is called a named graph [2]. An RDF store should have one default graph and zero or more named graphs.

RDF graph store providers do not have any explicit way to express any intention concerning access to data. In the paper we attempt to define the proper methods to find and modify the RDF data in a graph store with web

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service means. In this paper a new architectural style access to graph stores based on Representational State Transfer (REST) [3] is presented.

The paper is constructed as follows. Section 2 is devoted to related work. In Section 3, we propose a flexible solution for managing RDF graph store data. The paper ends with conclusions.

2. Related Work

Classical web services tools were focused on Remote Procedure Call (RPC), and as a result this style is widely supported. Unfortunately, it is often implemented by mapping services directly to language-specific functions calls.

The most popular protocol based on RPC style is XML-RPC [4]. It uses Extensible Markup Language (XML) in request body and response returned values. An XML-RPC message uses POST Hypertext Transfer Protocol (HTTP) method. Another RPC approach is JSON-RPC [5]. Its requests and responses are encoded in JavaScript Object Notation (JSON). A remote method can be also invoked by sending a request to a remote service using sockets or HTTP protocol.

Simple Object Access Protocol (SOAP) [6] is the successor of XML-RPC. SOAP is a protocol for exchanging structured information in the implementation of various web services. It relies on XML for its message format, and usually relies on HTTP, for message negotiation and transmission. It is also based on RPC style. SOAP has become the underlying layer of a more complex set of web services, based on Web Services Description Language (WSDL). Unfortunately, SOAP is very complex.

In the context of Semantic Web, there is also a new proposal for query processing and returning the query results service [7]. It describes a means of conveying SPARQL queries and updates from clients to query processors. SPARQL Protocol is described as an HTTP binding of abstract interface. It uses WSDL to describe a means for conveying SPARQL queries. Unfortunately, this protocol is dedicated only for graph stores which use SPARQL query language. Furthermore, this SOAP-based protocol is complex and can be significantly slower, because it usese verbose XML syntax. Additionally, this protocol disregards many of HTTP's existing capabilities such as: authentication, caching and content type negotiation. In contrast, here we do not use RPC style. We concentrate on defining mechanisms strictly dedicated to web graph stores, preserving the feature of using IRIs, Multipurpose Internet Mail Extensions (MIME) types, HTTP response codes, and thus allowing existing layered proxy and gateway components to perform supplementary options on the web such as HTTP caching and security enforcement. What is important is that our approach differs from the idea presented in [7] in that it does not depend on XML syntax or the complex architecture of [6].

3. RESTful graph store

In this Section we discuss the idea of using the RESTful access to graph store data. We define three aspects of RESTful graph store: (1) the set of query operations supported by the graph store, (2) the set of managing operations supported by the graph store and (3) the MIME of the data supported by the graph store and HTTP response status codes. Additionally, we introduce the ability to map the proposed IRI to SPARQL queries. We also outline the equivalent operations to more advanced queries.

3.1. Graph queries

In this Subsection we present query operations dedicated to graphs in the graph store. Let V be the set of all variables, then RDF triple pattern TP is a pattern in $(S \cup V) \times (U \cup V) \times (O \cup V)$ and V is infinite and disjoint from I, B, L and D (see Section 1). A variable is prefixed by "-" and the "-" is not part of the variable name. Triple patterns are started after the graph and are separated by "/". Elements of the triple pattern are separated by "|".

A Graph Queries Operation is an action that accepts some arguments A and transforms a graph store GS to another graph store GS': $OpGraphQueries_{GS}(A) = GS'$. Arguments should be in the RDF triples form, triple patterns form or empty set. The result is either GS' in case of correct execution or GS in case of error. These operations (Table 1) allow clients to manipulate RDF triples:

- $OpSelect(tp)$ with $\{tp : tp \in TP\}$. This combines the operations of projecting from the graph store.
- $OpInsert(t)$ with $\{t : t \in T\}$. This adds triples into the graph store.
- $OpUpdate(tp_1, tp_2)$ with $\{tp_1 : tp_1 \in TP\}$ and $\{tp_2 : tp_2 \in TP\}$. This can update triples from the graph store.
- $OpDelete(t)$ with $\{t : t \in T\}$. This removes triples from the graph store.
- $OpAsk(tp)$ with $\{tp : tp \in TP\}$. This tests whether or not a query has a solution.
- $OpDescribe()$. This returns a result containing data about graph store resources.

Table 1
RESTful graph store HTTP methods for graph update and query

HTTP Methods	Operations	Mapping to SPARQL
GET	$OpSelect(tp)$	SELECT
POST	$OpInsert(t)$	INSERT DATA
PUT	$OpUpdate(tp_1, tp_2)$	DELETE/INSERT
DELETE	$OpDelete(t)$	DELETE DATA
HEAD	$OpAsk(tp)$	ASK
OPTIONS	$OpDescribe()$	DESCRIBE

These operations should be executes with IRI defined in Augmented Backus-Naur Form (ABNF) [8]:
IRI = scheme "://" ihost "/" graph ["/" query]
scheme = "http" / "https" ; supported protocols
graph = "default" / ∗(iunreserved / pct-encoded / sub-delims) ; default or named graph
query = ∗ (iunreserved / pct-encoded / sub-delims) ; triples and triple patterns

Rules *ihost*, *iunreserved*, *pct-encoded* and *sub-delims* are defined in [9].

The IRI example of selecting triples from graph store: *http://example.org/default/-x|foaf:name|-name*. All graphs and triple patterns should be encoded in [9].

3.2. Graph management

A Graph Management Operation is an action that accepts some arguments A and transforms a graph store GS to another graph store GS': $OpGraphmanagementGS(A) = GS'$. Arguments should be in the IRIs form or empty set. The operation performs the described transformation of the graph store either completely or leaves the graph store unchanged. These operations (Table 2) allow clients to manipulate graphs:

- $OpList()$. Selects all triples from graph in graph store.
- $OpCreate(u)$ with $u \in I$. Creates a graph in the graph store.
- $OpLoad(u_1, u_2)$ with $u_1 \in I$ and $u_2 \in I$. Reads an RDF document and inserts its triples into the graph in the graph store.
- $OpDrop(u)$ with $u \in I$. Removes the specified graph from the graph store.
- $OpInfo()$. Returns information about graph store. It may display SPARQL Service Description [10]

Table 2
RESTful graph store HTTP methods for graph management

HTTP Methods	Operations	Mapping to SPARQL
GET	$OpList()$	SELECT * WHERE ?s ?p ?o
POST	$OpCreate(u)$	CREATE
PUT	$OpLoad(u_1, u_2)$	LOAD
DELETE	$OpDrop(u)$	DROP
OPTIONS	$OpInfo()$	*none*

These operations should be executed with IRI defined in ABNF:
IRI = scheme "://" ihost ["/" references]
scheme = "http" / "https" ; supported protocols
references = reference ["/" reference]
*reference = * (iunreserved / pct-encoded / sub-delims) ; IRI reference*
Rules *iunreserved*, *pct-encoded* and *sub-delims* are defined in [9].
The IRI example of listing triples from graph store: *http://example.org/default*. All graphs should be encoded in [9].

3.3. Media types and status codes

In this Subsection we discuss the body of request and response HTTP messages [11]. Supported MIME types can be represented by syntaxes, such as: Turtle [12], RDF/XML [13], RDF/JSON [14], or any other valid type. A request depends on an *Accept* header and a response depends on a *Content-Type* header. The response codes are presented in Table 3.

Table 3
Relationship between HTTP status codes and methods

Status code	HTTP methods	Response Contains
200 (OK)	GET, POST, PUT, DELETE	Serialized data?
204 (No Content)	POST, PUT, DELETE	Empty
304 (Not Modified)	GET	Empty?
400 (Bad Request)	GET, POST, PUT, DELETE	Empty or serialized error message
404 (Not Found)	GET, PUT, DELETE	Empty
409 (Conflict)	POST, PUT, DELETE	Empty or Serialized error message

3.4. Mapping to SPARQL

In this Subsection we show how to map SPARQL 1.1 [15, 16] clauses that are not showed in Table 1 and Table 2 to proposed operations. These mappings are presented in Table 4.

Table 4

Equivalent operations

SPARQL clause	Equivalent operations
DELETE	$OpUpdate(\varnothing, tp_2)$
INSERT	$OpUpdate(tp_1, \varnothing)$
DELETE WHERE	$OpUpdate(\varnothing, \varnothing)$
CLEAR	$OpUpdate(\varnothing, tp_2)$
COPY	$OpDrop(u);\ OpUpdate(tp_1, \varnothing))$
MOVE	$OpDrop(u);\ OpUpdate(tp_1, \varnothing);\ OpDrop(u)$
ADD	$OpUpdate(tp_1, \varnothing)$

4. Conclusions

The problem of how to adjust query to a graph store has produced many proposals. Most of them are hard to use without dedicated tools, hence making the problem seem difficult. We assume that the graph stores, to be more functional, should provide a simple mechanism to execute the queries. The main motivation for this paper is the lack of such requirements.

We have produced a simple, thought-out and closed graph store proposal. We believe that our idea is an interesting approach, because it is graph store independent. Our proposal can work either with mobile and other devices or a web browser and other software as a graph store client and server.

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