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FREE MARKET AND COMPUTATIONAL COMPLEXITY

Essays in Commemoration of Friedrich Hayek (1899-1992)

THE CHAIR OF LOGIC, INFORMATICS AND PHILOSOPHY OF SCIENCE
UNIVERSITY OF BIAŁYSTOK
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INTRODUCTION

The readers are presented the 18^{th} number of the annual set of „Studies In Logic Grammar and Rhetoric", which is the fifth consecutive volume complying with the joint thematic pivot. The methodology of social sciences and, in particular, its tasks resulting from the development of mathematics and informatics research constitute the above-mentioned pivot now.

The 'new' paradigm of social sciences methodology, even if it is still *in statu nascendi*, requires including the issues connected with undecidability, computational complexity or computational intractability in the set of tools and concept categories of empirical sciences. The above-mentioned problems developed on the ground of mathematics seem to relate directly to social sciences anywhere algorithmic methods of modelling or simulating of social processes are applied.

By means of this annual set we would like to celebrate the contribution of one of the most outstanding thinkers of 20^{th} century – Friedriech von Hayek, the creator of modern liberal philosophy – to the development of social sciences methodology in the 10^{th} anniversary of his death. Liberalism is, in his opinion, the only philosophy which is in agreement with the latest theories in the domains of physics, chemistry and biology, and especially with the science of chaos, formalized by Illya Prigogine. In free market economy like in nature, order arises from chaos; spontaneous action of million of decisions taken and million pieces of information received does not lead to disorder but to a certain higher order. Nobody is able to know, as Hayeck claims, how to plan an economic growth as we do not really know anything about the mechanisms governing the economic growth. There is a great number of decisions on the market that no computer, no matter how powerful it would be, could register and process.

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The question about the adequacy of constructive depiction of social phenomena, especially in the context of the decidability problem and algorithmic tractability of these phenomena, seems to be legitimate. Witold Marciszewski's essay, which opens the volume, describes the challenges for social sciences methodology in the context of complexity issues. In the first part of his essay the impact of logic and information technology on the current paradigm of social science is analysed. Then the degrees of complexity and their poor awareness in practice of social science are discussed. The author also presents the theoretical models as the example of modelling in social science as well as the concept of rationality. The starting point of the considerations is the issue of intelligence understood as rationality with inventiveness and which as a whole can be marked, in the author's opinion, as hypercomputational complexity.

Wiesław Banach presents the idea of self-organization developed by Friedrich von Hayeck, which constitutes the basis for his critique of the methodological assumption of constructivist rationalism. Hayek's idea of self-organization is the heart of his socioeconomics doctrine. The author points at two intellectual traditions of individualism in the European thought – an empirical or evolutionary tradition typical for Britain and a rationalistic French tradition which are the subject of Hayek's critique. For Hayek, it is the former tradition that bears the name of true individualism. The latter accounts for false individualism which leads to socialism or collectivism, i.e. inclined to oppose freedom. The historical context explains, as Banach shows, Hayek's critique of the constructivistic utopia and social planning.

However, the points criticized by Hayek were defended by some eminent authors, hence they are worth being presented to help a better understanding of Hayek's arguments. To some extent, this is done by Anna Zalewska who hints at some links between early mathematical economics, as represented by Pareto, and an appreciation of planned economy. Andrzej Malec, in turn focusses on the idea of central planning in its various versions to conclude that it is not necessarily confined to the doctrine of socialism.

Dariusz Surowik's paper is to hint at certain possibilities of mathematical modelling on economics with the use of a theory of rational choice; this is exemplified with the classical approach of Leonard Savage.

Tadeusz Tyszka's remarks provide some empirical support for the idea of self-organization and spontaneous order. The psychologists emphasize the fact that people have a limited capacity to process information. Consequently, human beings have to adopt a set of simplified rules of decision making. Additionally, an excess of information can deteriorate the quality of human

decision. Tyszka notes that the ideas of Hayek support perfectly such concepts of human mind describing the mechanism of its acting in the terms of data processing with is non-objective character.

The paper entitled "Mathematical methods on commodity exchange" concerns a case study prepared by Bolesław Borkowski and Arkadiusz Orłowski. The authors concentrate on some mathematical models and methods that can be applied to rational option pricing on commodity exchange. They present this mechanism with the example of Polish economical institutions, such as Poznań Exchange and Warsaw Commodity Exchange.

The last essay in this volume presents deep insights into the theory of chaos, which change a scientific way of looking at the dynamics of natural and social systems. Michał Tempczyk leeds the reader towards the main methodological assumptions of the theory, pointing at their applications in the field of social sciences.

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The materials published in the volume constitute modified versions of the invited papers, presented during 5th Workshop of Logic, Informatics and Philosophy of Science entitled *Free Market as a System of Information Processing. Some Issues of Algorithmization in Social Research*, organised by the Committee of Philosophical Sciences of Polish Academy of Sciences, the Departament of Logic, Informatics and Philosophy of Science at the University of Białystok, the Departament of Econometry and Informatics at Warsaw Agricultural University, The Institute of Philosophy at Jagiellonian University, the Adam Smith Centre with the support of the Faculty of Economics at the University of Białystok and the Białystok School of Public Administration.

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HYPERCOMPUTATIONAL VS. COMPUTATIONAL COMPLEXITY
A Challenge for Methodology of the Social Sciences

Motto One: *There are actually lots of threads that led to computer technology, which come from mathematical logic and from philosophical questions about the limits and the power of mathematics.*

Greg Chaitin[1]

Motto Two: *Computer simulations are extremely useful in the social sciences. It provides a laboratory in which qualitative ideas about social and economic interactions can be tested. This brings a new dimension to the social sciences where 'explanations' abound, but are rarely subject to much experimental testing.*

Richard J. Gaylor, Louis J. D'Andria[2]

1. The impact of logic and informatics on the current paradigm of the social sciences

1.1. The first term in the title of this essay, *hypercomputational*, requires elucidation as being quite a novelty (a bit shocking, perhaps) in the language of science. Fortunately, the term *computational* has a well-established meaning since Turing's seminar study of 1936. Fortunately, as well, it was the same Turing, in his work of 1938, who offered us a first hint toward the idea of the hypercomputational, the hint being involved in the concept of an *oracle* — a device (hypothetically postulated) to render values of uncomputable functions; such a rendering is now called hypercomputing.

Thus hypercomputational complexity is one which cannot be handled by algorithms, that is, computational devices. Nevertheless, it may be handled with other means. What means? Turing put forward his idea of oracle in

[1] "A Century of Controversy over the Foundations of Mathematics" in: C. Calude and G. Paun, *Finite versus Infinite*, Springer-Verlag, London 2000, pp. 75-100.

[2] From Introduction to *Simulating Society: A Mathematical Toolkit for Modeling Socioeconomic Behavior*, Springer Verlag 1998.
See www.telospub.com/catalog/FINANCEECON/SimSoc.html

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order to make the concept of intuition more precise, especially as appearing in the context of Gödel's discovery of undecidable statements in mathematics. If such a statement is acknowledged as true without any proof (and even without a chance of being proved), the human faculty acting there is what one calls *intuition* or *insight.* The same faculty is busy in judging mathematical axioms as true. Turing [1938] attempted at formalizing this informal concept of intuition.

More on this subject is to be said later. Here it is enough to express the conjecture that mathematical intuition may deal with the uncomputable. However, as being mathematical, it remains in the realm of numbers, while numbers are capable of being computed; if not computed in the strict Turingian [1936] sense, then in a way called *hypercomputing* – as suggested recently by a circle of researchers led by Jack Copeland.

Let us assume that at least some uncomputable functions can be hypercomputed. To make this a plausible conjecture, let us suppose there are magnitudes in the world which are both continuous and uncomputable. Next, suppose that such magnitudes can be correctly estimated by a cognitive apparatus (the senses, the brain) on the basis of analogue processing, that is, a kind of mirroring or copying, the copies being some states of the perceiving subject. As being true copies, they should involve magnitudes represented by continuous uncomputable numbers. Since they result from the process which operates on some magnitudes, they are somehow computed in an extended sense of the word. To avoid equivocation, it is reasonable that the difference between the computing in that broader sense and that in the strict (as in Turing [1936]) sense be rendered with a new term, such as hypercomputing.

The authors engaged in the information-processing approach to scientific research divide nowadays into the minority which claims the possibility of hypercomputing and the majority which rejects it. The name for the latter has been already coined, namely *computationism.* As to the former, no designation has been invented so far; it may seem misleading to use a term like "hypercomputationism" (as too similar in form to that so opposite in content), but no viable alternative is in view. To make the difference clearer, I emphasise it through adding the adjective "strict". Thus the opposition involved will be rendered as follows: **strict computationism** *versus* **hypercomputationism** — abbreviated as SC and HC, respectively.

1.2. Either point results in a specific methodological paradigm. Let us restrict discussion to methodology of social sciences. Computational methods enter a social science naturally in those parts in which phenomena can be

handled in a quantitative way, as in economics, a theory of social choice (votes are easily counted), and even in theories of social interactions as much as they are developed in a game-theoretical or similarily quantitative framework. At the same time, there is a strong trend from the very beginnings of sociology to shape it more on the image of historical investigations than that of natural sciences. And then enters the concept of understanding (German *Verstehen*) which appears in the technical term *understanding sociology*, going back to Max Weber.

There is an embarassing multitude of possible understandings of the word "understanding", but one interpretation is especially encouraging. It consists in resorting to the concept of intuition as discussed in philosophy of mathematics, and exemplified with such mental acts as asserting axioms, or judging importance of a mathematical problem.

The concept of mathematical intuition, in spite of its initial vagueness, has gained some preciseness owing to the theory of algorithmic complexity. After Gödel, Turing, Church and Post, nobody denies that there are acts of mathematical cognition which are not available for algorithms. Such acts, evidencing human creativity, are of two kinds. Some of them can do what algorithms cannot, the latter being too slow to solve a problem in a reasonable time (without a human prompt); thus human cleverness, when acting in a non-algorithmic manner, may succeed where a brute force fails. To account for such intuitions, it is not necessary to resort to the idea of hypercomputation. However, this may be necessary to account for human capability of finding axioms, including such ones as the famous Gödelian sentence.

Thus the social sciences owe much to mathematics, not only when taking models and algorithms from it, but also when facing the riddle of intuition. Once we follow the HC claim that mathematical axioms stem from intuition which may consist in hypercomputing, the same may be considered with respect to axiom-like propositions of social sciences (as, for instance, that concerning interaction between demand and supply, that stating an advantage of peace over war, etc).[3]

There is a price to be paid for such a help obtained from mathematics by social sciences. This requires endorsing a vision like that of Leibniz (preceded by Phytagoreans and Plato) according to which the whole reality is governed by numbers, as said in the Book of Wisdom: *omnia in numero disposuisti.*

[3] Ludvig von Mises [1966], using the phrase *a priori* instead of *intuitive*, claims that numerous statements of social sciences are apriorical, exactly in the way mathematical axioms are. This view is convincing, especially if axioms are construed as propositions of the kind called *meaning postulates* by Carnap [1958]; the occurrence of meaning postulates in empirical sciences is beyond any doubt.

This favourite maxim of Leibniz was also expressed in his saying: *Cum Deus calculat fit mundus*, and this agreed with his juvenile (in the earliest dissertation) idea: *essentiae rerum sunt sicut numeri.*[4]

However, why not perform a thought experiment and become a Leibnizian for a while? In this experiment, let us combine Leibniz's vision with the modern awareness of the computable, uncomputable and hypercomputable. Then a social scientist would deal with the world also (like the physical world) defined by numbers, and then the question would arise: whether in that world there are computable numbers alone, as believed by computationists (e.g. the physicist Ed Fredkin and the followers of his "digital philosophy"), or uncomputable numbers should be admitted as well, as believed by Penrose, Copeland and others?[5]

Even if there may seem something mysterious about such a philosophical framework, the strict-computationist alternative is mysterious as well. According to SC, what HC calls intuition also results from an algorithm, say Alg1. Nobody knows it, it has to be hidden somewhere in the interior of brain. At this point, one has to feel a touch of mystery. To wit, in those cases in which it has been proved about an algorithm that it cannot settle a question, SC has to assume, to avoid acknowledging intuition, that there exists a stronger algorithm (even if we know nothing of it) to perform the task too difficult for its predecessor. Only in such a way can SC avoid the verdicts of limitative theorems. Moreover Alg1 must be produced by something else, and that 'else' must be again an algorithm, say Alg2. And so on. As in the Catholic Church the principle "nulla salus extra Ecclesiam" (no salvation outside the Church), so in the SC epistemology there holds the principle "no cognition outside Algorithm".

Let us agree that SC and HC are equally justified as philosophical hypotheses which may be identified with what Popper calls *metaphysical research programmes.* Both have bright and dark sides. In fact, both provide a research paradigm which may prove fruitful, since in such a configuration negative results concerning any of them corroborate the other. Thus the best strategy is to carry out either programme, possibly, each by a different team. In what follows, the HC-vs-SC opposition will be exemplified

[4] The quoted sayings mean, respectively. as follows. *You [God] arranged all the things with numbers. When God computes, the world is becoming. The essences of things are like numbers.*

[5] Leibniz himself was divided between these opposite options. As one who constructed arithmetical machines, and fancied logical machines to solve all possible problems, he was like a modern computationist. On the other hand, his metaphysics of infinity and continuity would make him close to the opposite camp (that Leibniz's internal split is discussed by Marciszewski [1996a, 1996b]).

with presenting some eminent representatives of either conjecture in social sciences.

1.3. Among the pioneers who cleverly acknowledged the complexity in social systems, such as free markets, or democratic societies, there was Friedrich Hayek (1899-1992). His insights can be appreciated against the contrastive background of the views of Oskar Lange (1904-1965). The latter has a merit of a different kind — that of comitting a fruitful instructive error. Lange believed that the computational complexity of computer systems (as beeing attainable in the early sixties) can perfectly match the complexity of economic processes. It was the conviction on which he built his faith in the advantages of central planning over the *spontaneous order* (Hayek's expression) of free market and provided a relevant use of computers in planning.

However, the power of computing cannot be judged rightly without consulting mathematical logic from which the research in computational complexity issued (as recalled in Motto One). The latter is a tool in investigating complex systems, among which social systems display the uttermost complexity. When referring to mathematical logic, I mean those problems and results which are mainly due to Hilbert, Gödel, Turing and Post, to mention those most akin to computer science; close to them there are the achievements of Church, Tarski etc.

However, the limitative theorems that reveal the incompleteness of arithmetic and the undecidability of first-order logic were by many treated, up to some time, as little relevant to empirical research. People had believed that these theorems applied to some esoteric mathematics, and were not engaged in its applications, as modelling and digital simulation of social processes. Such an optimism – as to the chance of escaping computational limitations – is rooted in the following *Computability of the Empirical* assumption, which expresses the philosophical SC point.

[E-Comp] *All the relations holding in empirical reality* (1) *can be represented by computable functions and, moreover,* (2) *all of them can be calculated by algorithms which work with resources being within our reach.*

This claim is capable of being empirically tested and, possibly, falsified, at least in item 2. The core of Lange's contribution consists in stating such a falsifiable hypothesis; if falsified, it would confirm the contrary to it conjecture as defended by Hayek and the rest of the Austrian School. The claim E-Comp might seem plausible in the early sixties (the time of Lange's last publications), before the new science of computational complexity started to come into existence with contributions like those of Hartmanis and Stearns [1965]. Another instructive instance of disregarding complexity is found in the famous Club of Rome Report, based on computer simulations. A tone of such optimism sounds even in some recent views as that quoted in Motto Two (which can be endorsed, but only if supplied with due provisos).

This paper should (a) hint at some results which refute E-Comp(2), and (b) discuss the chances of E-Comp(1). Such chances would be challenged by a success of hipercomputation, that is, such performances in information processing would exceed the possibilities of Turing machines.

In the title of this paper, let me recall, the complexity to be handled by an insight (oracle) is called hipercomputational, while that being, at least in principle, capable of algorithmic approach is called computational. The latter notion comprises those cases which are computationally tractable, and the cases of intractability, that is, those in which algorithms require such big resources of time, memory, etc, that are practically useless (hence the proviso "in principle").

To sum up, there is a vital reason for which philosophy and methodology of the social sciences should be interested in hipercomputational complexity. With this concept there appears an opportunity to build a bridge between recent developments in informatics and the traditional theory of intuition, involved in sociology understanding. The concept of intuition becomes methodologically justified when (i) defined as complementary to the concept of algorithm (in the sense of logical complement), and (ii) proved non-empty. The latter is shown by the examples such as intuitive accepting of the Gödelian sentence as well as any mathematical axioms.

What may social sciences gain from that strategy, that is, treating sociological understanding by analogy to a non-algorythmic (hypercomputational) part of mathematical activity? Let us note that the most afflicting problem with understanding (or, intuition, etc) is its subjectivity and elusiveness. The problem is common, concerning the whole of knowledge, but it is only in the philosophy and methodology of mathematics where it is being efficiently investigated; there are thorough discussions concerning methods of an objective justifying of axioms (e.g. Maddy [1966]). These discussions put axioms beyond the domain of the computable, hence in the conjectured domain of the hypercomputable. Once we agree that some assertions in social sciences are like axioms in mathematics, we may apply those mathematical considerations to a better understanding of "understanding" in social sciences (cp. footnote 3).

2. Degrees of complexity, poor awareness of them in the practice of social sciences

2.1. The present section is to hint at the state of research in the social sciences which is far from being satisfactory from a methodological point of view. To prove such an opinion, some introductory remarks on complexity will be in order.

Mathematical logic led to the formalization of the notion of algorithm (Church, Markov, Post, Turing) as well as understanding that certain problems are algoritmically unsolvable. With the appearance of computing machines logicians started to inquire into practical capabilities and limitations of such devices, which resulted in the emergence of computational complexity theory from the logical theory of algorithmic unsolvability. Church's and Turing's study in the *Entscheidungsproblem* have demonstrated that checking whether a sentence has a proof is algorithmically unsolvable. (cp. Sipser). This result is, so to speak, infinitary in the sense that infinite time (measured with the number of operations) may be needed in the search for solution. What the theory of complexity is concerned about can be seen as a finitary version of the *Entscheidungsproblem.* Now we do not ask whether an assertion has (any) proof but if it has a *short* algorithmic proof ("algorithmic" can be rendered by "formalized" in Hilbert's terminology).

The short word "short" is crucial for defining degrees of complexity of a problem. *The longer is the shortest algorithmic proof needed for solution, the more complex is the problem whose solution is to be proved.* There is a method of distinguishing practically important intervals in such a scale, called *complexity classes*; an ordering of them yields degrees of complexity.

Now, as to hypercomputational complexity, it is beyond that scale, in the sense that only finite procedures are taken into account. However, in another sense, it can be viewed as being on the top (in that sense in which we speak that an infinite number is greater than any finite number). It is the latter sense to be referred to in the further discussion.

Those problems whose complexity is beyond any capabilities of computation are called *undecidable.* Those whose complexity would require inaccesible resources of time, memory, etc. for their algorithmic solution are called *intractable.*

When a research in social phenomena involves algorithms, it ought to be accompanied by the awareness of that conceptual framework, worked out in logic and computer science. In particular, a researcher should be aware of limitations of algorithm; only then he will be able to abandon projects which are unrealistic, and in the case of difficult ones find measures to make his project more feasible. Here are some questions to be considered with such awareness.

— [1] Do the algorithms needed to model and simulate social phenomena happen to have complexity which would make the issue involved an undecidable or intractable problem?

— [2] If so, are there any methods to transform the problem into being both tractable and duly approximating the answer required?

— [3] If so, these methods should be presented.

— [4] Are there in any social theory assertions not being justified by any algorithm?

The reply to [4] is obvious both for mathematical and empirical sciences. In a deductive mathematical theory such assertions are axioms, while in an empirical theory no algorithms are necessary to obtain an observational statement and meaning postulates (the latter in the sense given by Carnap [1956]). In spite of such obviousness, question [4] should be stated to create an opportunity for the next question, to wit:

— [5] What is the basis of accepting a statement in a theory if the acceptance is not substantiated by any algorithm?

When questions [1], [2] and [3] are put to physicists, one obtains clear answers accompanied by a list of examples of the problems being undecidable or intractable or else those having only approximate solutions. Such a state of affairs is nicely exemplified by Stephen Wolfram's paper *Undecidability and Intractability in Theoretical Physics* [1985].[6] When exemplifying undecidable and intractable problems, he takes advantage of the term *reducibility* which can be also rendered by *compressibility* as used in the theory of algorithmic information (Chaitin, Kolmogoroff). The lack of this property causes that an algorithm simulating the process in question has to reproduce it step by step, in an explicit simulation, without any possibility of shortening.

Irreducible computations may turn intractable because of the lack of time or space (memory). Also the undecidability appears as an usual phenomenon, as exemplified by undecidable propositions about the behaviour of the cellular automaton (CA). The occurrence of such undecidable propositions may be viewed as a consequence of computational irreducibility.

[6] Wolfram is widely known owing to his works on cellular automata, collected in [1994] and his being the author of software called "Mathematica". His monumental book [2002] claiming that cellular automata constitute an adequate model of physical world has become a scientific bestseller of the year.

Undecidability is found in physics in many areas what can by seen if physical systems are viewed as universal computers. There are many physical systems in which it is known to be possible to construct universal computers. "Apart from those modeled by CA — writes Wolfram [1985] — some examples are electric circuits, hard-sphere gases with obstructions, and networks of chemical reactions. The evolution of these systems is in general computationally irreducible, and so suffers from undecidable and intractable problems. [...] It is the thesis of this paper that such problems are in fact common. Certainly there are many systems whose properties are in practice studied only by explicit simulation or exhaustive search: Few computational shortcuts (often stated in terms of invariant quantities) are known."

2.2. It is worth while to compare a physicist's awareness of such limitations with the belief in unlimited power of computation characteristic of some social scientists. As if social phenomena had not been enormously more complex from those in the physical world. This criticism is directed against some projects enjoying an enormous prestige and influence. There is a lot of expert studies which bring important limitative results concerning the use of algorithms in social sciences (referred to in the next sections), but these have not even a fraction of the fame of those rather pretentious projects.[7]

Here are some examples of treating very complex problems as if they were easily tractable in an algorithmic way.

Example 1. • Strong AI (Artificial Intelligence). This is a project demanding to the highest degree, as it aims at a perfect simulation of the most complex entity in the whole Nature, namely the human brain. The accomplishing of that project would be of great consequence for social sciences since intelligent artificial agents could be organized into artficial societies (AS) whose behaviour would be fully predictable (as rsulting from the algorithms known to the constructors). This branch of computer science happens to be called *multi-agent simulation*. Not only AS is assisted by AI, the reverse holds too, since intellectual development of artificial agents depends on social interactions in AS, and such a feedback has to result in a monstrous complexity. In spite of that one hardly encounters messages (from the researchers involved) which would concern either undecidability or intractability of the problems being addressed.

[7] Negligencies of social scientists happen to be repaired by logicians and computer scientists who watch what is going on in social sciences and comment on it from a logico-methodological point of view.

Example 2. • Central socialist planning with the help of computers, in the current literature called *socialist calculation*, as mentioned above, was defended by Oskar Lange in his polemics with von Mises and Hayek. Their objections hinted at the enormous complexity of economic and other social phenomena, too big to be processed by the brains of planners. In the early sixties, Lange replied that what had been impossible before the inventing of computers, became viable and easy with their help. Even now this idea is defended by some leftist authors (e.g. Cottrell and Cockshott [1993]). Such a debate is likely to bring a conclusion, provided that Lange's followers will supply us with a realistic model of economy. Owing to that lucky feature of economic phenomena that they can be measured, it should be possible to estimate the order of magnitude of input data to be taken into account. Moreover, there are proposals as to mathematicsl models of the demand-supply equilibrium, economic development, etc. as created by Pareto, Lange and other eminent authors. The equations forming the model provide suitable algorithms; these considered with the magnitude of input data should give us an idea of computational complexity to be handled by computers employed by planners (provided that decidability is granted).

Example 3. • The Club of Rome Report of 1972 entitled *Limits to Growth* predicted a world-wide economic and ecological disaster after the end of the 20th century. That divination was backed not only by a respected group of intellectuals but also by the authority of computer science as simulations were carried out by MIT experts with the best then available machines. It was based on a simulation model, a mathematical representation of the main variables and their dynamic interactions known as the WORLD III model. "The forms of exhaustion predicted in the various scenarios simulated in the model start to emerge in the early twenty-first century, as the world population grows to a peak of 10 billion, per capita food production drops to a mere 15-25 percent of 1970 levels, pollution has risen tenfold, and the most important resources, such as oil and gas, have become depleted. Because of the so-called exponential character of growth and depletion, half-hearted or one-sided measures are of little avail. A drastic program of technological improvement such as energy conservation, for example, achieving 50 percent savings in 20 years against a background of, say, 2 percent growth in consumption, postpones the date of depletion by a mere 3 years." (See Van Dieren [1995, Introduction]).

When coming into the 21st century, one can easily judge the reliability of the message reported in Example 3. Let it be just recalled that after the collapse of socialism the food production in most post-socialist countries rose so dramatically that those being EU candidates are obliged to artificially cut production; otherwise they would be too competitive in the EU market.

The success of a model depends much on intelligent simplifications. Among the simplifications made in the Report there was the total omitting of the factors of scientific research and technological invention (there

is no need to comment if that was an intelligent simplifying). Obviously, such factors cannot be grasped in central economic planning. Even if the Laplacean demon revealed what is to be going on in the heads of future discoverers, the unimaginable complexity of each brain separately and still greater of their world-wide interactions would unavoidably hamper any computer-based predictions.

On the other hand, an intuitive understanding as expressed in axiom-like maxims, e.g. "the more economic freedom, the more economic information" may prove more reliable and more useful than the results of algorithmic procedures. Do such understandings result from some hypercomputational processes in our brains? This is an open question. They may be due to some algorithms which would be by far more efficient than "classical" algorithms based on logic of predicates, or probability theory, or else game theory. In the moment, having no way to settle this question, we should be glad for our ability to see the problem that by no means is trivial, and forms a considerable progress. To grasp it, one needs some messages which will be discussed in the next section.

Game-theoretical models and the concept of rationality

3.1. As an example of modelling in social sciences, let us consider the mathematical game theory, going back to von Neumann and Morgenstern [1944], which supplies social scientists with a standard model of rational human interactions. A game which became a standard paradigm of game theory, comprising a very large class of social processes, is called "prisoner's dilemma" because of the following story to exemplify the problem (cp. pespmc1.vub.ac.be/PRISDIL.html).

> Two criminals have been arrested under the suspicion of having committed a crime together. To obtain a sufficient proof in order to have them convicted, they are isolated from each other, and offered a deal: the one who offers evidence against the other one will be freed. If none of them accepts the offer, they prove cooperating against the police, and both will get only a small punishment because of lack of proof. Thus they both gain. However, if one of them betrays the other one, by confessing to the police, the defector gains more: he will be freed, while the other, remaining silent, will receive the full punishment (as one who did not help the police). If both betray, both will be punished, but less severely than if they had refused to talk. The dilemma resides in the fact that each prisoner has a choice between only two options, but cannot make a good decision without knowing what the other one will do.

"This simple game-theoretic model seems to capture in miniature something of the tensions between individual acquisitiveness and the goals of collective cooperation. That is of course precisely why it has become a major focus of modelling within theoretical sociology, theoretical biology, and economics. [...] It is no simplification to say that our strongest and simplest models of the evolution of biological and sociological cooperation–and in that respect our strongest and simplest models of important aspects of ourselves as biological and social organisms–are written in terms of the Iterated Prisoner's Dilemma." (see www.sunysb.edu/philosophy/faculty/pgrim/SPATIALP.HTM).

Originally the game was considered for two people playing one-shot version of the game (i.e., without iterations), but from a logical point of view (e.g. the point of decidability) the thing becomes interesting in iterated many-player games. At the same time, iterated games are in focus of a theory of social evolution as tending towards more cooperative behaviour. This is why they are worth to be studied. In a non-iterated game the most advantageous behaviour consists in acting selfish, that is, with the loss of the other player in order to maximalize your own gain. In an iterated game it is cooperation what proves more advantageous. The reason is that the increasing experience reduces uncertainty as to the partner's strategy; at the same time, they get opportunity to learn the advantages which in a long term results from cooperation. Thus the prisoner's dilemma yields a model of social evolution.

In studying evolution, a very efficient tool is provided by the theory of cellular automata (CA) (created by von Neumann, together with Stanis aw Ulam). Individual cells represent agents interacting with their neighbours (i.e. the surrounding cells) according some fixed rules, and changing their states (from a definite set of states) owing to interactions. As cells are rendered, e.g., by squares at a blackboard, each cell has eight neighbours. If the interaction rules are those involved in the prisoner's dilemma, then there are nine players. The number of strategies may exceed two (the cooperative and the competitive one).

Each player in such a display competes with each of its neighbors in an iterated prisoner's dilemma and totals its scores from those competitions. A player surveys its neighbours. If no neighbour has a higher local score, the player retains its original strategy. If it has neighbours with higher scores, it converts to the strategy of its most successful neighbour. The result is a model in which success is in all cases calculated against local competitors.

In the course of game, some strategies become more frequent than other ones, and in this sense they start to dominate; this evolution consists of

changes of configurations in the two-dimensional field. In some cases, it is possible to forecast the direction of evolution. Is it possible in each case? Is there an algorithm which in each case would tell us about the final result of evolution? That is: which strategy would preserve its domination? When translating the issue into a concrete example, one may ask whether peace and alliance between some former enemies will last for ever, or is to turn again into old animosity?

Grim [1997], who examined the case of dilemma as reported above, replies with the following conclusion: "There is no general algorithm [...] which will in each case tell us whether or not a given configuration of Prisoner's Dilemma strategies embedded in a uniform background will result in progressive conquest. Despite the fact that it is one of the simplest models available for basic elements of biological and social interaction, *the Spatialized Prisoner's Dilemma proves formally undecidable in the classical Gödelian sense.*" (Italics mine – WM.)

Thus, as to the example of competitors who become cooperative allies, when taking into account the complexity of real social situations, one has reasons to believe that such a case, so involved, is no more solvable than the relatively simple case investigated by Grim. That is to say, one cannot hope that any algorithm would settle the question, while one may hope that a clever politician would do. Analogously, no arithmetical algorithm does recognize the Gödelian sentence as true, but a clever mathematician does. Does this result form a process of understanding which in mathematical terms would mean a hypercomputing? Obviously, this is not a question to be settled at once, but (let me say it again) the dawning awareness of this question is a real achievement in exploring the limits of the human mind.

3.2. The prisoner's dilemmma forms an opportune framework to consider the notion of rationality which since the time of Max Weber (at least) has been vital for social sciences. Among the authors discussing the dilemma, there are two different uses of "rational". To put the difference in a nutshell, let it be roughly expressed with the following identities:

R1: rational = efficiently acting for self-interest;

R2: rational = able to correctly perform every computation.

Interpretation R2 may illuminate, by analogy or metaphor, the Weberian sense as found in the study of the Protestant ethics and capitalism, since the capitalistic rationality is related to precise calculations.

Let us turn to explaining both R1 and R2 in more detail. The former is defined as follows (see *Principia Cybernetica Web*).

> "The problem with the prisoner's dilemma is that if both decision-makers were purely rational, they would never cooperate. Indeed, rational decision-making means that you make the decision which is best for you whatever the other actor chooses." (pespmc1.vub.ac.be/PRISDIL.html)

This use of the term "rational", fairly frequent in the game-theoretical literature, should be treated as elliptic.[8] When fully articulated, the term should be replaced by the following:

> instrumentally rational (*zweckrational* – Weber's term) with respect to the goal being defined as the decision-maker's gain, irrespective of possible losses which his choice might bring to other beings.

This interpretation of the term "rational" does not imply the treatment as equivalent of these two philosophical maxims: *homo est animal rationale* and *homo homini lupus est*. Rationality does not consist in mere selfishness but in the ability to find out means for any goals, while in economics one's own profit is the goal considered.

The other concept of rationality in game theory, here referred to as R2, is related not to motivation but, most generally speaking, to information. With the unattainable perfect rationality there is contrasted the *bounded rationality*, that is, one which suffers some informational limitations, as incomplete information, absent-mindedness, limited foresight, limited reasoning capabilities, too small memory, etc.

This concept proves very fruitful for a game theory as it allows to solve mamy problems concerning optimal strategies. For instance, A. Neyman [1985] (who much contributed to the notion of bounded rationality) stated that if the size of memory (measured in the number of states in finite automata implementing strategy as players) is contained in the interval $[n^{1/k}, n^k]$, where n is the number of rounds, and $k > 1$, then cooperation is a profitable stategy for both players. Another example: there is a proof that cooperation is more advantageous than competition when the number of rounds is not known to players; this kind of uncertainty makes competition more risky (cp. Papadimitriou and Yannakakis [1991]).

[8] Such an elliptic use is manifest in statements like the following. "Indeed, social actors are not merely agents following rules in a strict way or pure rationalists maximizing a value. They also try to realize their social relationships and cultural forms." (Gomolińska [1999, p. 96). Take we this wording literally (instead of elliptically), then realizing social relationshios and behaving in a cultural way should be qualified as irrational.

In the final section which follows, which is to lead to the point stated in the title of this paper, it is the second game-theoretical concept of rationality which will be taken into account. We shall see how it contributes to the notion of intelligence.

4. Intelligence as rationality with inventiveness

4.1. The challenge to be met by the methodology of social sciences when faced with the theory of computational complexity may be rendered in the following reasoning.

(i) Computer simulation of social processes is possible then and only then if there is an algorithm for explanations and predictions concerning the process in question. (ii) Any theoretical explanations and predictions of social processes require taking into account intelligent behaviour of the actors involved. (iii) Hence, computer simulation of social processes requires an algorithm to simulate intelligent behaviour of the actors involved.

Thus the challenge will be met if one offers a definition of intelligence which would tell whether every intelligent behaviour could be simulated with an algorithm having a reasonable complexity. This is to mean that the answer in the affirmative will be substantiated if (*a*) an algorithm does exist and, moreover, (*b*) it enjoys a required feasibility; that is, the size of resources needed (as time and memory) is not as great as, say, exponential with respect to the size of input data. Correspondingly, the answer in the negative will be substantiated if either (*non-a*) no algorithm is attainable or, if it is attainable, (*non-b*) its computational complexity exceeds the limits of feasibility (tractability).

In what follows, I am to argue that the *non-a* situation is sometimes the case. The argument has to consider a theory of intelligence. A necessary feature of intelligence is one discussed in the previous Section, to wit rationality. This feature is defined by a set of criteria. Depending on a set of accepted criteria, we obtain one from among various notions of rationality, and in consequence we get one from among various notions of intelligence. Such a multiplicity is conspicuous in the case of bounded rationality, since differently defined bounds result in different concepts.

For instance, one may postulate that no rational being is ready to assert in one sentence that both A and non-A is true; but if a person asserts A in Sunday, and non-A in Monday, the process including both is not necessarily non-rational. However, we would not be so tolerant with respect to an entity supposed to be perfectly rational, that is, without any cognitive limits (like an omniscient God). Another example: we would hardly regard as rational a behaviour in which one does not grasp the validity of modus ponens reasoning. However, we would abstain from such a verdict in the case of a very long and involved reasoning; or in the case of calculating expected utility which would require an instant multiplication of, say, some 15-digit numbers.

Thus we obtain a generic concept (i.e., a necessary condition) for defining intelligence which yields the following proper inclusion: *every intelligent behaviour is rational*, that is, conforming to certain criteria of rationality. In this sense, if the criteria are suitably chosen, the behaviour of a machine can achieve a high level of rationality. Is it sufficient to call it intelligent? This depends on what is required more.

4.2. In search for another feature of intelligence, to complete the one discussed above, let us consider a connection between intelligence and life. Intelligence as the problem-solving ability is necessary to survive and to develop in a desired direction. Both survival and development require efficient problem-solving.

In such a context, it is easy to observe that living in a continously changing environment requires reacting to many unexpected situations. In such situations any once possessed *routine* does not suffice. Instead, one needs what is called *inventiveness*, that is, the capability of making *innovations*. The opposition of routine and innovation is crucial for this discussion. However, before we focus on it, let us look at the phenomenon of innovation in the broader context of evolutionary processes; such processes form a great part of what social scientists try to render with computer simulations.[9]

The accumulation of countless innovations leads to systems as intricate as market economies or democratic states. This course of events is embedded into an all-embracing process of social evolution driven by four basic mechanisms. To describe such a process (with hinting at these driving mechanisms) and its phases, we may start observation from any point of time whatever.

(0) At any point there is a set of strategies (or policies) that persist, that is, successfully reproduce themselves. This means the transmission of a program, or code, or set of rules, to the next generation of strategies. Such a persistence is accounted for by the basic inertia of all social systems. It is easy to notice its counterpart in a biological dimension, when strategies amount to well-tested ways of behaviour to serve the survival and development of a species.

(1) While some of these strategies are reproduced in a routine fashion, by copying, others will undergo change, e.g. chance mutation in the biological domain, or will be proposed as reforms by policy makers in response to certain problems. These are the sources of variation that introduce *innovation* into the set of strategies.

(2) Here a moral from the prisoner's dilemma proves to be in order. Innovation may disturb an established equilibrium of strategies; the ways of behaviour being safe so far become more risky because of the weakened orientation of agents in the altered environment. The actors may choose strategies either of conflict or of cooperation. In a long term, the latter proves more profitable for all the actors involved, they become the focus of effective alliances, and so the society ever better appreciates advantages of *cooperation*. It learns to coooperate. Such a course of events is more probable in free societies being, moreover, more advanced civilizationally. With others this requires a longer time and costs more, but eventually a cooperative strategy is likely to win everywhere; for, as Adam Smith put it, when accounting for what prompts humanity to save, there is the ever-present "desire for bettering our condition".

(3) However, an equilibrium so regained after the innovation had disturbed the previuos one, does not mean total peace and security. There is another factor, namely constant competition which is necessary for any development either in a biological (the Darvinian Struggle for life) and social dimension (economic competition, political elections). This feature of evolution is being summarized with the concept of *selection*. One should agree with Darvin that selection is a crucial factor for progress in evolution.

(4) As in every process of learning, the success of selected strategies amounts to *reinforcement* (that is reward, combined with punishment for non-selection). The so reinforced revised strategies are then diffused, via mechanisms of amplification, and transmitted via a system of inheritance, in successive generations of strategies (compare item 0).

4.3. It has been asserted above that two properties constitute intelligence: rationality (in certain defined bounds) and inventiveness. The argument is simply derived from the concept of intelligence as the ability of efficient problem solving. Obviously, problems are either expected or unexpected. Those expected are tackled through a routine which in most perfect form

[9] In the following discussion of evolutionary processes an extensive use is made of George Modelski's (1996) paper "Evolutionary Paradigm for Global Politics". In some places I quote his statements in a form similar to his original statements; however, I do not use quotations marks since adjustments to this text must have been made, thus departing from the literal wording.

becomes an algorithm, and this is the domain of rationality (as conceived by computer scientists, not necesarily by philosophers). Those unexpected must be handled through innovations, and this is the domain of inventiveness. It involves having new, even strange, ideas, and also the ability to put questions, to feel astonished, to discern what is important and what is not. There is no job for algorithms in such situations.

It follows, then, that intelligence consists of these two factors, each of them being necessary; taken together they form the sufficient condition for a behaviour to be intelligent. The rationality factor is predictable, the inventiveness factor is not. Now, to continue the argument concerning chances of the perfect computer simulation of social processes, we should address the question of how predictability is related to computability. Certainly, whatever is computable is predictable; there is no need to bother whether the reverse holds. For our argument this assertion is sufficient since it is equivalent with the assertion that non-predictability implies non-computability. Hence the inventiveness factor, as being not predictable, is thereby not computable, hence no subject to digital simulation.

In the evolutionary framework as sketched above, inventiveness plays the main role in the phase 1. It constitutes that part of evolutionary process which cannot have any computable mathematical model and any algorithm to be used in a digital simulation. This is not to mean that the method of simulation is useless in social research. On the contrary, the greater is the area of unpredictable, the more we should try to have a precise, certain, and as vast as possible knowledge about what can be known. This is what increases the chance of clever guesses.

Suppose that some uncomputable phenomena may be hypercomputable, that is, accessible for a non-algorithmic skill, like with an inventive researcher who finds out new axioms and new methods of reasoning. Suppose that the skill grows when the researcher wins more knowledge from computer simulations. Furthermore, consider that successful innovations in the world of algorithms are due to creative powers of intuition. Then the success of a cognitive enterprise requires a close cooperation between the algorithmic and the intuitive thinking. And then the challenge for methodology of social sciences consists in the arranging of a close aliance between these two powers in dealing with complexity of the social world.

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HAYEK: AN IDEA OF SELF-ORGANIZATION AND A CRITIQUE OF THE CONSTRUCTIVIST UTOPIA

The aim of this paper is to present an idea of self-organization developed by Friedrich von Hayek, which is a basis for his critique of the methodological assumptions of the constructivist rationalism. While being known in the literature of the field as a concept of spontaneous order or an idea of cultural evolution, Hayek's idea of self-organization is a heart of his whole socioeconomic doctrine. While abstracting from its assumptions, it is not possible to understand the core of Hayek's objections against the constructivist inclinations of any social planner who wants to reorganize a society on the grounds of new, utopian foundations.

The Concept of Spontaneous Order

While starting to reconstruct Hayek's concept of spontaneous social order, one must not omit a distinction introduced and repeatedly underlined by the author. According to this distinction there are two intellectual traditions of individualism in the European thought. On the one hand, one faces an empirical or evolutionary tradition typical of Britain. On the other hand, however, there appears a rationalistic French tradition. Yet, for Hayek it is the former tradition that bears the name of true individualism. The latter tradition accounts for false individualism which leads to socialism or collectivism, i.e. is inclined to oppose freedom. While not going into details, one may say it was R. Descartes's appearance that differentiated the traditions of individualism just mentioned. While underlining the individual's ignorance and poor capability for a conscious and intentional development of the civilization, the evolutionists criticized the Cartesian rationalism which filled the tradition of French individualism. They especially object to the engineering, constructivist declarations of the second part of *Discourse on*

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Method, and the author's magnified confidence in the power of human reason with the capital "R". An echo of Descartes's appearance was a skepticism and a contempt for any human achievement which was neither projected nor controlled by a human reason. In Hayek's opinion, such an attitude typical of the rationalistic individualists has unavoidably made them become the enemies of freedom. Their end is the organization, and freedom always means chaos.

This is why the most significant difference Hayek notices between British and French individualism amounts to various views on a role the tradition plays. At the same time the tradition is meant to be a process of transmission of the whole cultural heritage. For, according to the Cartesian idea, any useful solution or social institution is a work of human reason. It is the legislator equipped with an instrument of reason which constructs the whole society. Such an approach to the origin and development of social institutions has most frequently been expressed in the form of the idea of social contract which has particularly been appreciated by the French social philosophers. This approach presupposes that the institutions and practices are useful for people only if they have been developed consciously, perform a presupposed end and are fully controlled. The Cartesian "radical doubt" refuses to accept anything that cannot be logically inferred from clear and distinct premises. Yet, in this respect the British individualism declares for a respect for traditional customs, institutions, principles and practices whose origin and rational foundations are unknown to us. Both humility resulting from this and the role of tradition being stressed become, in Hayek's opinion, the most significant characteristic of the British individualism[1].

The existence of two intellectual traditions mentioned above, and of two opposite ways of perceiving the social reality resulting from these traditions, makes us distinguish, Hayek says, two sources and types of social order that are continuously present in the contemporary Great Societies. On the one hand, therefore, we face the arranged order which is of exogenous nature and described as a construction. On the other hand, there is the endogenous order which is described as a spontaneous order. This picture is followed by the remark that the ancient Greeks were happier than we are, for they used two simple words for these two types of order. The first is *taxis* which names the former type of order, and the second is *cosmos* to name the latter type of order. Hayek is going to borrow these words and apply

[1] F. A. Hayek, *Individualism: True and False, in: Individualism and Economic Order*, George Routledge & Sons, London 1948.

them as the technical terms to describe two types of order distinguished above[2].

While characterizing the spontaneous order Hayek states that this type of order is not willingly accepted by the researchers, and is of no interest to them. The reason is partially because these researchers cannot comprehend an order which is not being deliberately created, and partially due to the very fact that they take it to be always something aiming at a particular end. Such an approach is largely based on a conviction that there is no order not to be created by a man. This is because the spontaneous order (e.g. market order) does not stimulate our senses, but has to be chased by our intellect. We cannot see – or intuitively perceive in some other way – the order of ambiguous human actions. We are only capable of reconstruction of this order by means of the mind while tracing the relations that exist among its elements. Hayek appeals to the abstractness to call this very essential particularity of the spontaneous order. At the same time it is the level of being concrete by which Hayek defines the order being created. The spontaneous order contains a system of the abstract relations between the elements which are also and only defined by the abstract properties. This is why it cannot be intuitively perceived and recognized. The only way to let us know such an order is to appeal to the theory that allows one to make the nature of these relations clear.

While characterizing the spontaneous order in broad terms, Hayek gives a couple of examples where this order is present in the physical world, and also defines a degree and the conditions in which one is able to recognize and investigate it. In his opinion, there are many examples of the complex orders which we can bring about only if we apply the forces of which we know to make such orders' form. However, we can never contribute to the formation of this type of order through a deliberate placing every element in an appropriate position. For example, we will never produce a crystal or complex organic compound if we place particular atoms in the positions to form the lattice of crystal or the system based on benzol rings. Yet, what is important is that we can set the conditions for their formation in this way. To illustrate these claims, Hayek appeals to a school experiment where the iron filings on a sheet of paper are made to arrange themselves along some of the lines of force of a magnet placed below. Through this experiment we can predict the general shape of the chains to form as a result of the filings being combined with each other. However, we are not able to predict which

[2] F. A. Hayek, *Law, Legislation and Liberty*, vol. 1: *Rules of Order*, The University of Chicago Press, Chicago and London 1973, p. 37.

of an infinite number of lines finds the chains to determine the force of the magnetic field. This will depend on the position, direction, weight, roughness or smoothness of each of the iron filings and on all the irregularities of the surface of the paper. While impacting each other and the surrounding forces that emanate from the magnet and each of the iron filings are at any time to generate a unique example of the general model. The general nature of this model is to be defined by well known laws, but its concrete version always depends on particular conditions which are not to be fully known.

It should be strongly emphasized that when making comment on the example given above one ought to distinguish between the spontaneous nature of the order being generated and the spontaneous origin of the rules that are the basis for this order. In Hayek's opinion, it is possible (at least to imagine) that the formation of the spontaneous order is solely based on the rules that are a deliberate product of man. Despite this fact, however, such an order would still be described by the author as the spontaneous order. One can, of course, doubt that while the rules being the basis for this order are implemented as a result of conscious and intentional decisions, the order is taken to be the spontaneous order by Hayek. On the other hand, one has to admit that one must not take the spontaneity too broadly. It means that one should not identify this quality with an unlimited elementality. What is meant is the elementality which is not subordinated to any rules or principles. If this were the case, one would not talk about the spontaneous order. Instead, one would talk about the perfect disorder, i.e. the chaos. For, unlike the disorder, any order is described by means of the regularity, and the regularity is nothing more than keeping the rules or subordination to the rules. What distinguishes the order from the disorder is then the elements of the former displaying some regularity, i.e. *ordering* according to some rules.

Of course, in his conception of the spontaneous order Hayek focuses on the issue of its existence and manifestation in the social world. The only thing we can actually do in favor of the spontaneous order being manifested in a society is setting the general conditions that stimulate the formation of such an order. Therefore, the most suitable situation is to set the optimum conditions for the individuals to use freely *the knowledge of the particular circumstances of time and place*. In other words, the situation in question should allow the free individuals to act within the limits of the rules of law[3].

[3] F. A. Hayek, *The Constitution of Liberty*, The University of Chicago Press, Chicago 1960, p. 220.

The order of the market is a manifestation of the spontaneous order in which Hayek is especially interested. In his opinion, the necessary condition to understand the core of the market is to set free from fallacious suggestions that associate the market with the economy. In the strict sense of the term, the economy consists of the complex actions through which the particular resources are allocated on the grounds of competitive ends and by their importance, according to a fixed plan. Therefore, a typical economy is a household, a farm or a company. For in these sites one is to realize the particular hierarchy of ends while disposing the particular resources. The order being consciously and intentionally arranged (*taxis*) is then the economy. However, the market order is not, and could not be, in Hayek's opinion, managed by such simple balancing of ends. On the contrary, it provides the particular participants with the particular and incommensurable ends which are recognizable only by those who realize them. The market is a typical example of the spontaneous order (*cosmos*), i.e. the order that realizes no hierarchy of ends. To avoid an undesirable effects of the confusion mentioned above, Hayek postulates that we should accept a separate term to describe a system of numerous and mutually connected economies, which constitutes the order of the market. The term that most adequately represents the core of the market order is, in Hayek's opinion, "catallactics". It is derived from Greek verb *katallattein* which meant not only "to exchange", but also "to admit into the community" and "to change from enemy into friend". This is how in Hayek's terminology catallactics has become a tool to define what he takes to be the market order[4].

In the later period of his activity, while being influenced by new, developing disciplines and scientific theories, and also due to the very fact that his terminology was not commensurate with these theories, Hayek was more and more frequently keen to apply the idea of cultural evolution to express his view on the social order being spontaneously arranged. As he claimed, the idea of cultural evolution and the idea of spontaneous order were twin notions. Therefore, to synthesize various ways of manifestation of the spontaneous order (e.g. in the sphere of free market, law, politics, language, morality, etc.), he decided to reflect on the problem under discussion on the grounds of the idea of cultural evolution and the theory of culture. In other words, Hayek expresses the idea of self-organization by means of the idea of cultural evolution, so that he shows how the origin, development

[4] F. A. Hayek, *Law, Legislation and Liberty*, vol. 2: *The Mirage of Social Justice*, The University of Chicago Press, Chicago and London 1976, p. 108-109.

and success of cultural institutions – being an unintended effect of human actions – determine the social coordination of the individuals. Hayek's idea of cultural evolution was most fully expressed in the epilogue to volume 3 of *Law, Legislation and Liberty*. While taking position on the discussion about the sources of human values Hayek, who was eighty years old at that time, tried to present his views clearly, for he had no time to complicate the issue. He rejected the foundations of sociobiological theories and presented the culture as a third, and most important, source of human values: "Culture is neither natural nor artificial, neither genetically transmitted nor rationally designed. It is a tradition of learnt rules of conduct which have never been invented and whose functions the acting individuals usually do not understand" [5].

Hayek repeatedly emphasized that the idea of cultural evolution was undoubtedly older than the idea of biological evolution, and Darwin applied in biology Mandeville's and Hume's findings in the field of social thought. Although both the cultural evolution and the biological evolution are based on the principle of selection, the former is not to be associated with concepts like: natural selection, struggle for existence or capability to survive by the best adapted. Such concepts have been imported from biology by social Darwinists who thus missed very important element, i.e. a selective evolution of rules and practices. Hayek, who was sorry for this misuse, tried to highlight the core of the difference of both ideas. He says that in the social evolution a selection of physical and hereditary individual properties is not the decisive factor. What counts is a selection through imitating the institutions and habits which most effectively operate. Although it is also reinforced by the successes of both the individuals and the groups, it does not result in the hereditary property of the individuals. Instead, it does result in the ideas and skills, i.e. the whole cultural heritage which we pass through learning and imitation. In Hayek's opinion, this heritage is composed of our customs and talents, tools and institutions, that is, all what is a result of the adaptation to a previous experience to be collected due to a selective elimination of less appropriate practices.

[5] F. A. Hayek, *Law, Legislation and Liberty*, vol. 3: *The Political Order of a Free People*, The University of Chicago Press, Chicago and London 1979, p. 155.

The Problem of the Use of Knowledge as a Core of the Economic Problem of a Society

Prior to a discussion on a critique of the foundations of constructivism-oriented social planners, one should define the way in which Hayek comprehends knowledge present in a society. In his opinion, it is the epistemic problems (which are linked with getting, passing and applying knowledge within the society) that determine the planner's difficulties and make his projects turn into utopia. The core of Hayek's position on the issue of knowledge was formulated during a dispute over the rationality of the socialist economy. While bypassing superfluous details of the dispute, and especially its course (which impacted the evolution of his views), one should appeal to a famous article of 1945 titled *The Use of Knowledge in Society*, which is unambiguously the most characteristic text on the problem of knowledge of this period. The very first sentence of this article makes a reader have no doubt as to what the key issue of the dispute over the rationality of socialist economy is. In this article he directly asks: "What is the problem we wish to solve when we try to construct a rational economic order?" [6] When we presuppose we have got a complete knowledge of the available means, are able to state a particular system of preferences and possess a complete, required information – we face a purely logical problem. This is the problem of optimizing (*a trial of getting the best result through the medium of the given means*), which can be formulated mathematically, i.e. the problem to be usually faced by an engineer. What is a typical characteristic of the problem raised in this way is always that the response to the question about the best way of utilization of the available means is *implicite* included in our presuppositions. However, the problem of such kind is not, as Hayek states, the economic problem which the society faces. This becomes fully understandable when one realizes that the so-called "data" which are the starting point of the economic calculus of the whole society do not and can never get "dated" with a single mind. A particular nature of the problem discussed "is determined precisely by the fact that the knowledge of the circumstances of which we must make use never exists in concentrated or integrated form but solely as the dispersed bits of incomplete and frequently contradictory knowledge which all the separate individuals possess. The economic problem of society is thus not merely a problem of how to allocate *given* resources – if *given* is taken to mean given to a single mind which deli-

[6] F. A. Hayek, *The Use of Knowledge in Society*, in: Ch. Nishiyama, K. R. Leube, *The Essence of Hayek*, Hoover Institution Press, Stanford 1984, p. 211.

berately solves the problem set by these *data*. It is rather a problem of how to secure the best use of resources known to any of the members of society, for ends whose relative importance only these individuals know. Or, to put it briefly, it is a problem of the utilization of knowledge which is not given to anyone in its totality"[7]. It is not the scientific knowledge, i.e. knowledge in the form of the general rules, whose importance Hayek emphasizes, but *the knowledge of the particular circumstances of time and place.* An individual who has knowledge of the details of the environment in which the individual acts gains an advantage over all other individuals, for (s)he has got a unique information that can successfully be utilized. In short, what is the most essential characteristic of the sort of knowledge under discussion is that this knowledge cannot be passed to any central power.

According to what has been written above, what becomes obvious is Hayek's demand to leave making the economic decisions to the individual who knows the reality of the environment. The capability of the optimum utilization of knowledge, and of the solution of economic problem of the society, is present only in the community of free individuals who are left to decide on their matters. There is no other social and economic system to be capable of the immediate adaptation to the changeable conditions, and thus of ensuring as appropriate allocation of resources as the market system requires (where the choice of ends and means is made on a micro level). Therefore, one should leave the capability of the final decision on individual matters to the individual. This would allow the individual to immediate adaptation to the changeable conditions. This is the only way to allow for bringing out and utilizing a potential present in the society. The discovery and the skilled utilization of this potential is as important as the utilization of natural resources, or of scientific or technical achievements.

Hayek's Critique of the Constructivist Utopia

While presenting Hayek's critique of constructivism one should approach his way of understanding the subject of this critique. In Hayek's opinion, constructivism is a symptom of a magnified belief in the power of human reason with the capital "R". It is based on an utopian view that all the useful solutions and social institutions are (and should be) a product of a deliberate project of the social planner. More precisely, to make these solutions and institutions useful for people, one has to create them consciously and to realize the end presupposed, and also to allow for taking a full control over them. As it has been written, in Hayek's opinion, the inventor of the constructivist approach was Descartes and especially the heirs to him who continuously made misuses while adopting the settlements of the father of the rationalist philosophy to the field of social sciences. However, the constructivist aspirations are not the matter of previous centuries, so that they are of an interest only to a historian of idea. As a witness of the 20th century's events, Hayek does not doubt the constructivist tendencies have not only remained vital, but have also gained a new support, and seem to indivisibly dominate in contemporary economic, political and social life.

Hayek writes that as a result of domination of the constructivist tendencies in the 20th century, we have decided to replace an anonymous and impersonal market mechanism with a collective and conscious making ourselves as well as all the social forces direct towards the ends picked up deliberately. This shift means, in his opinion, an entire separation from the individualistic tradition which had formed the Western civilization, and amounts to getting on "the road to serfdom". The terms like "political engineering" or "social engineering" have become fashionable catchwords, which express the fascination with a "conscious" control over the whole of the social world. What is always the end of a social planner's constructivist measures is a transformation of the spontaneous order into the organization, of cosmos into taxis, or of catallactics into the economy. This shift was caused by the very fact that during the first half of the 19th century the meaning of the term "science", which had previously been understood broadly, got narrower and narrower and became a synonym with the terms used to name the natural (i.e. physical and biological) disciplines. These disciplines started to demand a special rigor and certainty for themselves, and their success has led to an unknown before fascination over the methods used by them. These methods have thus been imitated and applied in the social sciences. It is this unreflective imitation and transfer of the methods and solutions of the natural sciences to the social sciences which Hayek calls the scientism, that is a source and a basis of the contemporary constructivists' aspirations. Although in a later period Hayek softened his antinaturalistic position a little, he remained an irreconcilable critic of the scientism so defined[8].

[7] Ibidem, 212.

[8] F. A. Hayek, *The Influence of the Natural Sciences on the Social Sciences*, in: *The Counter-Revolution of Science. Studies on the Abuse of Reason*, Liberty Press, Indianapolis 1979, p. 19-20.

Objectivism, Collectivism and Historicism as a Basis for the Scientism Assumption

To try to understand the core of Hayek's critique of the constructivism, one should appeal to his objections to the objectivism, collectivism and historicism, respectively. According to Hayek, these three positions characterize the scientistic attitude the most while being the cause of all errors and misunderstandings linked with this attitude. To simplify the whole issue a little, one might say that while criticizing the constructivism broadly defined, Hayek directs his most significant objections towards its tools. The objectivism is the first position to be analyzed by Hayek's critique. While defining it he points out that in the research both on man and on society the objectivism expresses most characteristically in various trials to free from the subjective knowledge we have. The objectivism states that the social reality can be described in the objective language, i.e. a language independent from the individual's subjective knowledge. While criticizing this Hayek says that the things which appear before us as identical may not at all be identical or similar in any objective sense, i.e. may possess no common but subjective properties. In other words, the qualities that we perceive are not any properties of some objects, but they are the ways in which we have learnt to group or classify the outer stimuli. What we call the social facts are not anything given in the objective way, which means they cannot exist independently from the acting agents' consciousness. Moreover, for the objectivity-oriented researcher who usually applies the quantitative methods there objectively "exists" only what is measurable, countable, and what defines a constant relation between the measurable magnitudes. In short, what "exists" is what can be defined and expressed in terms of mathematics. At the same time, what cannot be confirmed by numbers and does not contribute to make the reflection on the social reality more scientific is sentenced to mockery or oblivion. In Hayek's opinion, "the blind transfer of the striving for quantitative measurements to a field in which the specific conditions are not present which give it its basic importance in the natural sciences, is the result of an entirely unfounded prejudice. It is probably responsible for the worst aberrations and absurdities produced by scientism in the social sciences" [9].

A methodological collectivism is the next position characteristic of the scientistic approach. It is closely linked with the objectivism and most frequently expresses through the tendency to take the collections like society, economy, capitalism, industry, social class or state to be the data. More precisely, all these collections are meant to be the objects of which one may formulate the laws due to their observation as the wholes. Hayek criticizes the collectivism and states that while the physical phenomena may be directly observed (they are directly given to us), and, therefore, such a natural scientist's attitude is fully justified, the social phenomena are not observable in this way. This is why a social scientist has to start from the theoretical settlements which are based on the subjective knowledge gained by an appeal to the analogy to the scientist's own mind: "Social wholes are not given to us as what we may call *natural units* which we recognize as similar with our senses, as we do with flowers or butterflies, minerals or light rays, or even forests or ant heaps" [10]. While trying to show the collectivistic approach advocating naivety, Hayek emphasizes the complex wholes, which are the basis and subject of their research, are nothing more than the provisional theories to be present in general consciousness. These wholes are the models which to explain the connections between particular and separate phenomena being observed. Instead of reconstruction of the wholes on the grounds of the connections between individual minds directly known, the methodological collectivists attempt to grasp these wholes as if "at once".

The third methodological assumption to serve the description of the core of the scientistic approach is the historicism. In short, this attitude amounts to a trial of making history a "science" which would remain the only valid reflection on the social phenomena. For Hayek, the critique of the historicism is the best way of picturing his antinaturalistic position on the status and the methods of the social sciences. In his opinion, the so-called historical facts cannot be defined in terms of time and space coordinates. On the contrary, any trial to define them has to take a form of a mental reconstruction, i.e. a model composed of the individuals' understandable attitudes. The mental reconstruction of a historical fact is based on the same foundations the reconstruction of any other social fact is. In short, this reconstruction is a theory, and allows one to understand the social phenomena and processes (or past phenomena and processes as in case of history). The course is not, Hayek says, the first one which states "given" historical facts to use them for later formulation of the generalizations about them. One's research procedure in the field of history goes in the very opposite direc-

[9] F. A. Hayek, *The Objectivism of the Scientistic Approach*, in: *The Counter--Revolution of Science*, op. cit., p. 89-90.

[10] F. A. Hayek, *The Collectivism of the Scientistic Approach*, in: *The Counter--Revolution of Science*, op. cit., p. 96-97.

tion: first one formulates the theory to use knowledge of a given period to select the elements which compose the historical fact in an understandable way. Moreover, "the naive view which regards the complexes which history studies as given wholes naturally leads to the belief that their observation can reveal *laws* of the development of these wholes. This belief is one of the most characteristic features of that scientistic history which under the name of historicism was trying to find an empirical basis for a theory of history or a *philosophy of history*, and to establish necessary successions of definite *stages*, *phases*, *systems* or *styles* following each other in historical development" [11].

To summarize the critique of the scientism, Hayek confronts the planner's constructivist problem with the engineer's technical problem. While accepting the assumptions given above, the social planner makes the scientistic misuse. He thus approaches the social issues in a way the engineer approaches the technical problem he is to cope with. In Hayek's opinion, the engineer fully controls a piece of the reality in which he is interested. It is hard to talk about any misuses or erroneous assumptions of the engineer's activity until this activity refers to "his world", i.e. the world being described by the objective regularities of nature. (A huge technical progress or the successes of the natural sciences may serve as a practical confirmation of this very fact.) However, when the engineer or anybody who accepts the engineering assumptions and research perspectives attempts to transfer and apply his "tools" to the social world (and especially the field of economy), this must produce a negative result. The engineer who becomes a social planner or a "social engineer" begins to face the problems whose nature is by no means compliant with the problems being previously solved. The core of the misuse linked with the transfer mentioned above can be reduced to an erroneous presupposition according to which the objected reality to be comprehended and expressed in the measurable magnitudes is a target of the social planner's actions. The consequence of this presupposition being accepted is a belief that the planner as well as the engineer has (or may have at any time) a full knowledge required to successfully fulfill his intentions. To gain this knowledge, and to state all the information and conditions of actions, is for the planner only a technical problem to be easily solved. In other words, the planner presupposes that he has already got all the "data" required to solve the problem. At the same time, the knowledge of all that is called "data" may only exist in a dispersed form. It is accessible only

[11] F. A. Hayek, *The Historicism of the Scientistic Approach*, in: *The Counter--Revolution of Science*, op. cit., p. 128.

for the individual who acts in his native environment while taking the form of *the knowledge of the particular circumstances of time and place*. Most frequently it is the practical knowledge called "knowledge of how" which usually cannot be verbalized or articulated. What is an immanent feature of such a kind of knowledge is, first of all, that no one can pass it to any central power. This is why it cannot become a basis for any contstructivist techniques. The planner talks about the "society knowledge" and therefore, amounts to nothing but using a metaphor.

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FROM THE GENEALOGY OF MATHEMATICAL ECONOMICS: WALRAS, PARETO AND LANGE

All models are wrong, only some of them are useful.
William Edwards Deming

1. Mathematics allows us to solve various complicated issues concerning numerous aspects of life. Are mathematical methods universal enough to be used to study the dynamism of economic life? Economic life has both its quantitative and qualitative aspects and its phenomena are highly complex and changeable. The subject of research being relatively stable forms of the occurrence of economic phenomena and the search itself concerning calculation of objectively existing relations among different quantities, it is undoubtedly possible to achieve the desired results through certain mathematical methods (e.g. appropriate algorithms and techniques). However, the scope and the sort of methods to be applied to economic aspects, where the qualitative aspects or mechanisms of the examined economic problem are not well enough known or recognized, remains a question. How strong should the assumptions be for the mathematical method to strike equilibrium? What is the relation of abstract and generalizing models to existing reality? Nicolas Kaldor ([2]) claimed that *any theory must be based on all sorts of abstracts, however, the abstracts must not be selected in a vacuum space: they must be related to the characteristic features of economic process we experience.* Irrespective of an answer to the above questions, it may be stated that modern economy is highly mathematical. Growing formalization of economy introduces to the existing theory new methods (e.g. set, game or chaos theory) thus extending its scope and precision and comprising more and more aspects of economic life (such as analysis of the problem of choice in uncertainty conditions). Despite the fact that modern economics is unquestionably abstract and thus hardly representative for the existing world, it has managed to develop a number of practical mathematical techniques, such as linear, web, dynamic or genetic programming.

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2. From the historical point of view, mathematical methods have always played a bigger or smaller part in economic consideration. Italian monetarists in their works (B. Davanzatti – 1588 and G. Ceva – 1711) already postulated and attempted at wider use of mathematical methods in economics. Since late XVII c. attempts at employing mathematics for economic consideration purposes have become more frequent. The following ought to be mentioned:

- C. Beccaria – 1765 – consideration concerning pathology of economic life based on calculation and algebra,
- D. Bernoulli – 1781 – applying theory of probability to calculating the chances of gaining certain profit,
- N. F. Canard – 1801 – attempt at arriving at mathematic formula to determine demands and purchasing capabilities of buyers,
- J. H. Thunen – 1826, 1850 – consideration carried out partly in mathematical form,
- A. Cournot – 1838 – mathematic expression of correlations between demand, supply, prices, costs and incomes under different competitive conditions and monopolization degrees.

The last of the above-mentioned is considered to be among the first pioneers of the so-called formalistic revolution in economics, i.e. a precursor of general application of mathematics to the analysis of economic phenomena. Acknowledged as a great mathematician, he in a way anticipated Walras' ideas. Embracing correlations within economy, he claimed that in order to fully and rigorously solve the problems concerning respective parts of economic system, it is necessary to take into consideration the entire system. Cournot might have felt that the mathematical analysis was not adequately developed at the time. The reason that he did not deal with complex correlations of the theory of overall equilibrium may have been the numerous assumptions that had to be made to analyze the problem. Or he simply may not have had enough courage to deal formally with such a complicated problem. As it is, the first step was taken by Walras, who modeled a system of overall equilibrium through a clear and concise formal description with the use of mathematical notation.

3. **Walras Leon Marie Esprit (1834-1910)** with his 1873 work *Elements d'Economie Pure ou Theorie de la Richesse Sociale* ([6]) gave rise to the so-called Lausanne School in political economics. Another representative of this school was **Vilfredo Pareto (1848-1923)**, Walras' disciple who continued working on his conception and later succeeded him to the chair at Lausanne University.

Walras' interests included mainly overall equilibrium of goods on the market assuming that economy possessed self-driving mechanisms that restored the equilibrium of economic system upset by external stimuli (such as change in technology or consumer tastes). He focused on the exchangeability of economic goods. He studied the relations between demand and supply given a price that ensured their equality and lead to equilibrium. Walras believed that, since those relations concerned quantities, they had best to be presented through mathematical equations. Thus, knowing such parameters as for example consumer tastes or production technology, it is possible to mathematically establish optimum quantitative proportions among respective elements of economy, i.e. determine economic equilibrium. What takes place spontaneously through adjustment trial-and-error processes can be calculated with the use of algorithms provided that as many equations can be created as there are unknowns.

Let us assume that:

- economy consists of two sectors: enterprises and households,
- enterprises do not buy goods from one another,
- household preferences do not change,
- the level of technology does not change,
- there is a state of full employment,
- all industrial branches remain competitive.

Households are characterized by certain preferences and limited income. They enter the commodity market expressing a demand for goods offered by enterprises. Prices and amount of goods are established on this market. The market is in the state of equilibrium when the amount of particular goods offered and demanded is equal. On the market of manufacturing factors the situation is reversed. Enterprises make demands on households for production factors. According to supply of production factors by households, the prices of these factors are established. Equilibrium is struck when the offered amount equals the demanded amount in relation to each production factor. Households achieve their income on the market of production factors and spend it on the commodity market. The flow of income between enterprises and households represents the national economic income. For it to strike equilibrium, households must spend all their income. Enterprises, based on prices (of goods and production factors) and accessible technology, manufacture products in a way that maximizes their profits (a particular product ought to be manufactured with minimum costs and maximum profit). After a longer period of time, competition forces will lead to a situation where the price of goods will equal average production costs. To balance the level of national income, enterprises have to spend on production factors market all they acquired on commodity market. It should be mentioned that respective parts of the abstract model of economy are interrelated: a change in price of one product leads to repercussions in entire system.

Let us introduce the following symbols:

1) Let us assume that we have
   - H – number of households
   - F – enterprises
   - n – products
   - m – production factors.
2) Products:
   - x^h – vector of household h demand for products
   - x^f – vector of products offered by enterprise f
   - p – vector of product prices.
3) Production factors:
   - v^h – vector of production factors offered by household h
   - v^f – vector of enterprise f demand for production factors
   - w – vector of production factor prices.
4) Private property:
   - π^f – enterprise f income
   - σ^{hf} – part of enterprise f income held by household h.

Each enterprise will be in the state of equilibrium if:

$$\max \pi^f = px^f - wv^f$$
$$\Phi^f(x^f, v^f) = 0$$

i.e. if it maximizes its profit, on condition that together with the growth of amount of produced goods increases a demand for production factors.

Each household will be in the state of equilibrium if:

$$\max U^h = U(x^h, v^h)$$
$$px^h = wv^h + \sigma^h \pi$$

i.e. if the usefulness function representing household h preferences and its supply of production factors is maximized, provided that the cost of goods purchased by household h equals the sum of income acquired through selling production factors and the part of enterprise profits that belongs to household h.

Commodity market will be in the state of equilibrium if

$$\sum_{h=1}^{H} x_i^h = \sum_{f=1}^{F} x_i^f \qquad \text{where } i = 1, \ldots, n$$

i.e. if the sum of all products purchased by households equals the amount of all products manufactured by enterprises.

The productionfactors market will be in the state of equilibrium if

$$\sum_{h=1}^{H} v_j^h = \sum_{f=1}^{F} v_j^f \qquad \text{where } j = 1, \ldots, m$$

i.e. if the sum of all production factors offered by households equals the sum of all production factors purchased by enterprises.

According to Walras and Pareto, the overall economic equilibrium can be assessed through such equations if only the number of those equations equals the number of appearing variables. Thus, as far as the number of equations is concerned, we have:

- $nF + mF + F = (n+m+1)F$ equations in the enterprise equilibrium system
- $nH + mH + H = (n+m+1)H$ equations in the household equilibrium system
- n equations in the commodity market equilibrium system
- m equations in the production factors market equilibrium system,

which gives us a total of $(n+m+1)(F+H)+n+m$ equations. Since one of them will be dependent, the number is reduced by one to $(n+m+1)(F+ +H)+n+m-1$ equations. As far as the number of unknowns is concerned, we have:

- $n+m+1$ variables on the part of h^{th} household (x_i^h – products where $i = 1, \ldots, n$; v_j^h – production factors, where $j = 1, \ldots, m$; μ^h – Lagrange's multiplier), which, the number of households being H, gives us $(n+m+ +1)H$ unknowns
- $n+m+1$ variables on the part of f^{th} enterprise (x_i^f – products where $i = 1, \ldots, n$; v_j^f – production factors where $j = 1, \ldots, m$; μ^f – Lagrange's multiplier), which, the number of households being F, gives us $(n+m+ +1)F$ unknowns
- $n+m$ variables on the part of price equilibrium (p_i – product prices where $i = 1, \ldots, n$; w_j – production factors prices where $j = 1, \ldots, m$)

which gives us a total of $(n+m+1)(F+H)+n+m$ unknowns. Since one of the goods is taken as unit of account (*numeraire*), their number is reduced by one to $(n+m+1)(F+H)+n+m-1$ unknowns.

Thus, the number of equations equals the number of unknowns. However, Pareto himself recognized the calculations to be complicated. Let us assume that we deal only with 100 households, 70 enterprises, 700 products and 3 production factors; in such far from realistic case we would have to solve 120 382 equations.

4. Pareto's contribution ([5]) consisted mainly of extending the application of mathematical methods, developing the concept of overall equilibrium and on reformulating the idea of usefulness. Pareto observed that usefulness is a relative value (which is more useful – one or two glasses of beer? – it depends on, for example, whether we are thirsty or not and whether it is warm or cold). Thus, he parted with the theory of usefulness measurability (which underlined previous thinking in Lausanne School) and based what became the so-called theory of choices on certain data observable in one's management.

Pareto used the notion of indifference curves formulated by another economist, F. Edgeworth which visualized the scale of consumer preferences in relation to a given pair of goods. A consumer can acquire those goods in different quantitative combinations. Indifference curves are not measured. They are based on the choice one makes overcoming the difficulties in satisfying one's tastes. The choice is made in certain conditions on the basis of rational incentives in accordance with the economy principle. Needs are approached praxeologically. Tastes are identified with needs. As a result, when satisfying one's tastes, one makes choices according to preferences scale which, as Pareto initially believed, is statistically estimable. Indifference curves visualize consumer preferences scale. Respective indifference curves bring together all the possible combinations representing the same level of needs satisfaction. The higher the curve, the higher the level of needs satisfaction. Consumer possibilities determined by income are presented in the so-called price path. This method is based on the idea that desires (i.e. consumer preferences and possibilities) can be presented in two-dimensional space and that they visualize one's actions determined by external factors. Similar reasoning applies to a producer who decides on a certain combination of production factors. Given the indifference and production curves (visualizing the quantitative combinations of goods lost by the producer in relation to the gained ones), Pareto created a price theory where there is equilibrium between tastes and obstacles (between consumption and production), i.e. a price theory that, according to him, ensures overall equilibrium. Pareto tried to extend Walras' theory of overall equilibrium to the sphere of politics. In his studies he considered the actions of economic units not only in case of free competition but also of monopolization, such as, for example, socialist economy.

5. Walras' and Pareto's ideas greatly influenced the shape of other economists' viewpoint. For example, the thesis that socialism was theoretically and practically capable of rational income sharing was accepted by the ma-

jority of economists between the thirties and seventies. Neither Mises (it is impossible under the socialist system to rationally share income, since if there exists no free market, there are no independent prices, which makes it impossible to make rational decisions concerning income sharing) nor Hayek (the solution of the problem of income sharing in socialist system is possible theoretically but not practically, since socialist planners would not be able to gather enough necessary data, let alone the need to solve a system of millions of equations connected with the problem) were able to effectively convey their critical comments concerning socialism. In fact, some of those comments concerned Walras' equilibrium model on which economic analysis was based back then, in particular, its static character which did not explicitly allow for any individual enterprise activity and which did not deal formally with an adjustment processes in case of upset equilibrium.

Oscar Lange (1904-1965) ([4]) also refuted Hayek's arguments proving that it is possible to apply a market mechanism to socialist economy, which would lead to solving simultaneous equations through an empiric procedure of trial-and-error. The starting point of Lange's considerations is any given price system. When demand exceeds supply, prices are increased, whereas when supply exceeds demand, prices decrease. Final equilibrium is gradually achieved through such a trial-and-error process (first described by Walras). The prices satisfy the system of linear equations. Lange assumed that the process is convergent and goes in the direction of a price equilibrium system.

The market mechanism and trial-and-error process suggested by Lange served as a calculating apparatus for solving a system of equations. The solution was found through a convergent iteration process. Iteration worked on the basis of feedback which was supposed to gradually eliminate deviations. Lange imagined the process as a certain mechanism which (thanks to back coupling) automatically eliminates disturbances. According to Lange, such a mechanism stimulated market functioning whereas the market itself was one of the oldest recognized tools of solving simultaneous equations. Fascinated by electronic mathematical techniques, he wrote in 1965: *What is the problem? Let us make the computer solve simultaneous equations system and we will have the results in less than a second. Market process and trial-and-error equation procedure turn out to be old-fashioned. As a matter of fact, they should be treated as specific pre-electronic era calculating apparatus* while simultaneously expecting of these techniques the following kind of help: making it possible to change and re-schedule a plan, preparing alternative plans that would be adjusted to various objective situations that may crop up, and enabling the choice of optimum plan.

6. Economic models are idealistic and simplified forms of reality. They are for us in order to better understand complicated processes of economic life and to achieve certain goals (e.g. decision making). The question remains to what degree they comply with existing reality. Walras' and Pareto's mathematically expressed models gave rise to the so-called mechanistic paradigm in the analysis of economic processes. This paradigm, rooted in the mechanistic vision of the world, assumed as the theoretical basis of economy ideas borrowed its form classical mechanics. Such an approach is nowadays questioned in a number of basic assumptions ([1],[3]):

- *the analysis of economic processes concerns the state of equilibrium*; when monitoring real processes, it is obvious that economic development never reaches the state of equilibrium (cases of reappearing innovations) but only heads in the direction of the state which, on the other hand, constantly changes;
- *the knowledge of the process is full, which allows to make optimum choices*; limited calculative capacities and time limits force the decisions to be made based on the simplified models of reality and be far from optimum;
- *the only competition is the price competition which acts gravitation-like leading towards the state of relative equilibrium*; competition is a form of rivalry, it is won by those economic entities which manage their resources most effectively;
- *the criterion of economic entities activity is maximum profit*; real-life decision-making processes show that maximization possibilities are very limited if knowledge is limited: apart from trying to maximize profits, one also strives to ensure further development of the enterprise;
- *time is an absolute (the way it is understood in Newtonian mechanics): enterprises always remain in the state of equilibrium and the results of decision making are immediate*; in real-life processes the equilibrium time is usually much shorter that time of transition from one state of equilibrium to another; transition periods are of great importance.

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POSSIBLE LEVELS OF CENTRAL PLANNING

The aim of the paper is to discuss some limitations of planning in the area of economic activity. To achieve the goal we analyze the notion of planning and some of arguments for and against central planning. Most of the ideas of the paper are inspired by the essay of G. Warren Nutter – "Central Economic Planning: the Visible Hand" [1].

Free market *versus* central planning

Free market economy is commonly seen as the system of spontaneous order in which autonomous selfish actions of individuals constitute a socially accepted result. As Adam Smith wrote in his "The Wealth of Nations"

> By pursuing his own interest [an individual] frequently promotes that of the society more effectually than when he really intends to promote it [2].

On the other hand, at least from the beginning of the 20th century the idea of rational improving of this spontaneous order has been a permanent feature of our social thinking. As G. Warren Nutter recites:

> And so planning has been advocated to increase economic efficiency, reduce unemployment, control inflation, moderate the business cycle, distribute income more justly, make the economy grow faster, make it grow slower, prevent discrimination, eliminate pollution, improve the quality of life, and so on. In other words, planning is frequently hailed as a cure for whatever seems to be the economic ailment of the moment [3].

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1 G. Warren Nutter, "Central Economic Planning: the Visible Hand" in: G. Warren Nutter, "Political Economy and Freedom. A Collection of Essays", Liberty Press, Indianapolis 1983, pp. 107-129.

2 G. Warren Nutter, op. cit., p. 113.

3 G. Warren Nutter, op. cit., p. 113.

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In the following two paragraphs we examine whether the above opposition – free market versus central planning – is entirely correct.

Planning – a reconstruction of the notion

In our discussion we assume that the notion of planning involves four elements – a starting point, a final point, a set of rules of transformation, a time of transformation. Having this assumption, we define a plan as the idea of **how** to **achieve** a **goal** (which is a **future situation**), from a **present situation** in a **given time** using only **rules** from a **given set** of possible (or accepted) rules of transformation.

We can symbolize the above in the following way:

$$S_m = F_m(S_n, R_1, R_2, \ldots, R_k, T_w),$$

where

S_m, S_n are respectively a final point and a starting point (S_m, S_n belong to a given set of possible situations),

$R_1, R_2, \ldots, R_k$ are rules of transformation ($R_1, R_2, \ldots, R_k$ belong to a given set of accepted rules of transformation),

T_w is a time given for the transformation (T_w belongs to the set of time periods)

and

F_m is a plan of transforming S_n into S_m in a time T_w by the proper use of the rules $R_1, R_2, \ldots, R_k$ (F_m belongs to the set $\mathbf{F}$ of all possible arrangements of given rules of transformation).

The set of all possible situations achievable from a given starting point in a given time by the use of the rules of a given set of rules of transformation is **the range of possible planning** connected with given starting circumstances. In symbols:

$$RPP(S_n, R_1, R_2, \ldots, R_k, T_w) = \{S_x : S_x = F_x(S_n, R_1, R_2, \ldots, R_k, T_w)\},$$

where x is a variable.

It is obvious that it is not possible to achieve any final point we like having defined a starting point, a set of rules of transformation and a time for transformation (if the "power" of rules is limited and the given time is limited). However, it may be questioned whether every two sets of rules of transformation determine different sets of final points having defined a starting point and a time for transformation. In symbols:

$$\{R_1, R_2, \ldots, R_k\} \neq \{P_1, P_2, \ldots, P_k\} \rightarrow$$
$$\rightarrow RPP(S_n, R_1, R_2, \ldots, R_k, T_w) \neq RPP(S_n, P_1, P_2, \ldots, P_k, T_w).$$

In terms of our discussion it is the problem whether free market rules and central planning rules can lead us to the same results in the same time from the same starting point.

Levels of central planning

Having the above definition of planning and assuming that central planning is the planning performed by the government, we see that the opposition free market *versus* central planning is not entirely correct. Every government plans at least at the areas of tax policy, monetary policy and government expenses policy (i.e. expenses connected with financing the army, the police, the government itself etc.). In fact, we haven't got just two opposite models: free market or central planning. It is better to talk about a spectrum of levels of central planning – with the minimal central planning on the one side of the spectrum (centrally planned: tax policy, monetary policy, expenses on the army and the police), trought various kinds of soft central planning (centrally planned: as above plus expenses on social policy) and various kinds of medium central planning (centrally planned: as above plus a system of governmental indirect impact on economic decision of individuals; such system is a system of certain legal regulations: progressive taxation, tax reductions, licensing regulations for certain economic activities) and with several systems of strong central planning on the opposite side of the spectrum. The systems of strong central planning are based on the principle of direct governmental impact on every particular activity. One of these (strong) levels is exemplified by the system of central planning commonly imputed to the command economy of the Soviet Union:

> For example, one commonly held view portrays Soviet economic programs as a set of boxes within boxes. According to this view, the first box to be built is the one corresponding to the five-year plan, and even this one is said to have its general contours determined by a long-range plan, covering fifteen to twenty years. Once the five-year box is constructed, the next step is to fit successively smaller boxes inside it, each applying to a successively smaller time period. Thus, the schedules of day-to-day activity contained in quarterly and monthly plans are visualized as mere miniatures of the grandest scheme of all[4].

[4] G. Warren Nutter, op. cit., p. 111.

So, free market economy is also a subject of central planning. However, the central planning of free market economy is never based on methods of direct governmental impact on every particular activity[5].

That is why we tell that the opposition free market *versus* central planning is not entirely correct. The correct question is as follows: which level of central planning is the best from the point of view of economic progress? And in particular: can any system of strong central planning be more effective than free market economy?

Some arguments for the idea of strong central planning

The issue of the optimal scope of central planning has been discussed for centuries and some of the most important arguments for and against the idea of strong central planning were put forward more than two hundred years ago within the discussion on the mercantile system then dominating Europe:

> Mercantilism was, of course, nothing more than the economic side of the authoritarian state. It was common for government to regulate all kinds of economic activity: to fix prices, wages, and interest rates; to prohibit speculative trading; to specify the quality of goods; to license labor; to prescribe what people should and could consume; to create monopoly rights for favored proprietors; to control chartering of corporations; to foster state enterprises; to control foreign trade; and so on. The American colonies revolted against this excessive governmental meddling in economic life as much as anything else[6].

Nevertheless the issue was not solved at that time and several economic systems with at least elements of strong central planning have been carried into effect since then. Among countries which have had the experiences of this kind we can recite not only the Soviet Union and countries of the so-called former "Soviet camp" but also such countries as France, Norway, the Netherlands, Sweden, Japan, the United Kingdom, Korea, Taiwan and India[7]. Also in the United States the issue of instituting central economic planning was discussed in 1970s[8].

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[5] Free market system can be well defined as the system of economy based on two principles: the principle of private property and the principle of liberty of economic activity.

[6] G. Warren Nutter, "Strangulation by Regulation" in: G. Warren Nutter, "Political Economy and Freedom. A Collection of Essays", Liberty Press, Indianapolis 1983, p. 87.

[7] G. Warren Nutter, op. cit., p. 114.

[8] G. Warren Nutter, op. cit., p. 107.

In my opinion, among all arguments for the strong central planning there are two which are the most important.

The first argument is that the interest of no individual is the same that the interest of the society as a whole. So, the society needs to regulate all kinds of economic activity to secure the interest of the society as a whole and to prevent the society from being hurt by selfish actions of individuals.

The second argument is that no individual has the full knowledge about the economic environment. In particular, no individual has the full knowledge about the economic plans of the others. That is why no individual can prepare a completely rational plan for her economic actions. So, the government should help individuals in their economic activity by establishing central plans of economic actions. As Vera Lutz wrote in her "Central Planning":

> Collective forecasting (...) is supposed to make a twofold contribution to the solution of this problem. The first is that of rendering the economy "transparent", by gathering together and making generally available the knowledge, beliefs and intentions (often referred to for short as "information") of the individual economic agents regarding future developments in their respective sectors. The second is that of making economic activity "coherent", be welding the individual forecasts and plans into a consistent whole, corresponding to a "common view of future economic development"[9].

Some arguments against the idea of strong central planning

Among the arguments against strong central planning which were raised upon last two centuries the following arguments are, in my opinion, the most important.

The first: it is not only a peculiarity of individuals to make errors in plans of economic actions, the governmental central plan also can be wrong and the consequences of such situation are rather unpleasant:

> That is, the likely mistake inherent in a centralized forecast will have a more harmful impact on the economy than the variety of mistakes distributed among individual forecasts, since the very spread in the latter, involving overlapping margins of error, generates differential market adjustments that diminish average forecasting error over time[10].

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[9] G. Warren Nutter, op. cit., p. 117.

[10] G. Warren Nutter, op. cit., p. 119.

The second: since the interest of no individual is the same as the interest of the society as a whole, any central plan is merely a resultant of interests of various individuals and groups involved in preparation of the plan.

The third: the plan itself may carry into effect the unwelcome situations it predicts. So, to avoid such unwelcome situations the responsible government should probably – if necessary – forge the plan. It was the case of the Fourth Plan in France:

> (...) the Fourth Plan took no account of repatriation from Algeria because, as the commissioner of planning explained, "it was impossible to build a Plan on such a disagreeable eventuality. The government might have been reproached for having precipitated the event by announcing it"[11].

The fourth: the process of preparation and monitoring of any central plan costs a lot. So, if superiority of strong central planning over softer forms of central planning is not proved, it is not reasonable to spend money on strong central planning. In other words, softer forms of central planning are much cheaper.

Inventiveness – the strongest argument against the idea of strong central planning

Recently one more argument against the idea of strong central planning is raised and, in my opinion, it is the strongest argument: the idea of strong central planning cannot be reconciled with the idea of inventiveness. Any invention is always a product of creative thinking and so it is always unforeseeable. If our social activity cannot be foreseen it cannot be also the subject of effective strong central planning.

In the terms introduced above, we say that – thanks to the inventiveness of the mankind – the set of possible (acceptable) rules of transformation continuously grows larger. So, even if the starting point of a plan could be described as precisely as we want, it is not possible to make an effective plan since the set of rules of transformation is not constant. Every new invented rule changes **the range of possible planning** connected with given starting circumstances.

[11] G. Warren Nutter, op. cit., p. 120-121.

Moreover, the grater invention is made, the more important changes in the range of possible planning it involves.

However, it is obvious that central planning of a certain – lower than strong central planning – level is inevitable in the economic activity of the mankind.

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LEONARD SAVAGE'S MATHEMATICAL THEORY OF DECISION

Certain elements of the mathematical theory of decision were developed as early as at the end of the 19th century, or even earlier. The theory of probability and its applications to gambling were also highly developed. The concept of maximization of expected utility had already been introduced, however, it was not used for a long time. Still, the mature mathematical form of the theory of decision has been developed only recently. In the 1940s and 1950s a few works were published influencing greatly the development of mathematical research connected with the theory of decision, including *The Theory of Games and Economic Behaviour* by John von Neumann and Oskar Morgenstern (the theory of games) and *The Foundations of Statistics* by Leonard Savage (axiomatic foundations of the theory of subjective expected utility). These works formed the axiomatic bases for the contemporary theory of decision, and the results logically expanded in a number of theoretical findings from the beginning of the 20th century.

Probability

In the 1900s economists noticed that probability is a useful theoretical tool for modelling such phenomena as financial investments or decision-making in companies. However, the notion itself was ambiguously interpreted those days. The most famous interpretations are: classical, frequentist (objective), logical, and subjective[1].

[1] Three of these interpretations are still being used today.

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Classical (Bernoulli, Laplace)
The probability of an event is the ratio of the number of "favorable" cases to the total number of cases, where the cases are equally likely.

Frequentist
The probability of an event means the frequency of its occurrence in a great (potentially infinite) number of repeated trials. This interpretation is a basis for common statistical methods of testing hypotheses.

Logical (Keynes)
Probability is connected with statements and can be deduced from truth-value of the premises of the statements for which it is being inferred. (However, it remains unclear how the logical value of premises is defined.)

Subjective[2] *(Savage, Finetti)*
Probability is a subjective degree of conviction, which could be attributed to any event, either repeated or not. It could be measured by psychometric methods, such as observing choices in gambling. This approach was criticised since two people using the same information may disagree as to the probability of an event. The concept of subjective probability has its shortcomings. From a mathematical point of view, the sum of probability of event A and its opposite should be one. However, this in not the case in the concept of subjective probability. Some researchers, e.g. Serik Suleimenov, claim that in order to avoid this problem it is essential to abandon the notion of subjective probability and apply special functions to objective probability.

Utility and preferences

At the beginning of the 20th century the idea of utility as a psychological term was notorious as it was doubtful when used to calculate the probability of events. Therefore, economists started emphasising the notion of preference as a primary psychological notion. An individual may not be able to attribute utility expressed in the number to an object but presumably can say which of the alternatives he prefers or decide that all the alternatives are equivalent to him. Thus, it seems that preference as a qualitative binary relation can be perfectly perceived as a primary operation, on which the axiomatic theory of decision can be built.

[2] The presentation expounds on the concept of subjective probability as it is the basis for Savage's theory of decision.

The theory of decision

Leonard Savage formed axiomatic bases for statistics, which combined the theory of inference with the theory of decision. According to his theory, it was possible to pose and try to answer the question:

Given specific data, what decision to make?

In his theory Savage employed the subjective interpretation of probability and followed Ramsey and von Neuman and Morgenstern in using preference as a primary psychological notion. The basic notions of his theory are acts and consequences, which are used to define the notion of decision. Thus, we say that a decision has been made when there was a choice between two or more acts. Deciding on a fact involves considering consequences that can be inferred for any possible state of the world.

The following exemplifies the notion of an act and a consequence:

Example[3]

Your wife has just broken five good eggs into a bowl when you come and volunteer to finish making the omelet. The sixth egg lies unbroken beside the bowl. For some reason it must either be used for the omelet or wasted altogether. You must decide what to do with this unbroken egg.

You must decide among three acts only. Namely:

the first act: to break an egg to join the other five eggs,

the second act: to break an egg into a saucer for inspection,

the third act: to throw an egg away without inspection.

We have the following acts and consequences:

[3] The example is taken from Savage's *The Foundations of Statistics*, p. 14.

Act	State [4]	
	Good	*Bad*
break into a bowl	*six-egg omelet*	*No omelet, and five good eggs destroyed*
break into a saucer	*six-egg omelet, and a saucer to wash*	*five-egg omelet, and a saucer to wash*
Throw away	*Five-egg omelet, and one good egg destroyed*	*five-egg omelet*

If two acts have the same consequences in all states of the world, there is no reason to view them as two separate acts. Thus, acts can be identified with sets of their consequences. Formally, an act is a function that attributes consequences to every state of the world.

The set of all acts available in a given situation will be marked as F (in the omelette example set F consits of three elements).

In a set of acts the relation of preference is introduced. Considering two acts, an individual may prefer act f to act g. In other words, if an individual has to decide between f and g, and no other acts are involved, he will choose f.

An individual cannot simultaneously prefer act f to g and act g to f. That is why Savage replaces the realtion *is preferred* with the relation *is not preferred.*

Formal notation

$s, s', \ldots$ – states of world,
S – set of states of world,
A, B, C, ... – events (subsets of set of states of world),
$c, c', \ldots$ – consequences,
C – set of consequences,

[4] States of eggs correspond to states of world in this example.

$f, g, h, \ldots$ – act[5] (functions from set S into set C). (If f denotes act, and s denotes state of world, thes $f(s)$ denotes the consequence assigned to state s.)
F – set of acts.
$\succcurlyeq$ – relation of preference on set F.

A constant act, is an act, whose consequences are independent from the state of world[6]. A formal definition of the constant act is as following:

Def. 1.
f ($\in F$) is a constant act if and only if for any $s \in S$ holds $f(s) = c$, where $c \in C$.

F^{const} will be used to denote a set of constant acts.
By $[f, A; g, A']$ we denote an act h, such that, for $s \in S$:

$$h(s) = \begin{cases} f(s), & \text{if } s \in A \\ g(s), & \text{if } s \in A' \end{cases}$$

Savage introduces a relation of preference, when some event is given. The definition is as follows:

Def. 2
$f \succcurlyeq_A g$: if $[f, A; h, A'] \succcurlyeq [g, A; h, A']$ for a certain $h \in F$.

$f \succcurlyeq_A g$ is understood as: act f is prefered to act g, when event A is given.
Savage also defines the notion of null event.

Def. 3
An event A *is null* if $f \succcurlyeq_A g$ for any $f, g \in F$.

Postulates of Savage's theory of decisions[7]

P1. *$\succcurlyeq$ is complete and transitive.*

P2. *For any $f, g, h, h' \in F$, not null event $A \subseteq S$ holds:*
$[f, A; h, A'] \succcurlyeq [g; A; h, A']$ *if and only if* $[f, A; h', A'] \succcurlyeq [g, A; h', A']$.

[5] An act is viewed as a particular attribution of consequences to the states of the world and corresponds with real or hypothetical alternatives that a decision-maker chooses from.
[6] Savage insists on the occurrence of all constant facts, which is problematic as it is possible that the act "Boeing 707 is flying over the Atlantic" is required while all oil sources are used up.
[7] The notation of axioms used here is not identical with Savage's original notation.

P3. *For not null event* $A \subseteq S$, *and for any* $f, g \in F^{const}$:

$$[f, A; h, A'] \succcurlyeq [g, A; h, A'] \text{ if and only if } f \succcurlyeq g.$$

P4. *For any* $A, B \subseteq S$ *and for* $f, g, f', g' \in F^{const}$ *such that* $f \succ g$ *and* $f' \succ g'$:

$$[f, A; g, A'] \succcurlyeq [f, B; g, B'] \text{ if and only if } [f', A; g', A'] \succcurlyeq [f', B; g', B'].$$

P5. *There exist* $f, g \in f^{const}$ *such that* $f \succ g$.

P6. *For any* $f, g \in F$ *such that* $f \succ g$ *and for any* $h \in F^{const}$ *there exists finite partition* $\mathcal{P}$ *of the set* S *such that, for any* $H \in \mathcal{P}$:

i) $[h, H; f, H'] \succ g$,

ii) $f \succ [h, H; g, H']$

P7. *For any* $f, g, h \in F$, *if* $f(s) \succ g(s)$ *for any state* s *of event* A, *then for any* h

$$[f, A; h, A'] \succcurlyeq [g, A; h, A'].$$

Savage's axioms resemble those of von Neuman and Morgenstern's theory of games. They both assume that the ordering relation is complete and transitive[8]. Both theories contain the *Sure Thing Principle*, which means that common elements in any pair of alternatives can be ignored or eliminated (axiom P2). Savage's system requires also some other special axioms, the most important of which are axioms P3 and P4. They help to achieve subjective probability from subjective utility.

Axioms P5, P6, P7 are mainly technical conditions (*non-triviality, continuity*, and *domination*). They are mostly non-controversial.

The advantage of Savage's theory is the fact that it does not assume *a priori* the existence of subjective probability but derives it from axioms connected with preferences. Moreover, the objects of preference in this theory are concrete and easily identifiable with the elements of real decision problems.

The drawback of Savage's theory is the fact that it allows a set S to be an infinity set. Most theories alternative to Savage's theory, which preserve the linear order and additive subjective probability and which use subjective probability on finity sets, use the notion of lottery. It is achieved by either the change of the set of consequences C for the set Pc (all the lotteries in C) so that acts attribute lotteries to the states of the world, or by forming mixed acts as lotteries whose results are acts in the meaning of Savages's theory[9].

In practice, in the case of any real application of the theory, the set of the states of the world and the set of consequences can be identified only for the finite number of elements. This was why Savage made a difference between *small worlds*, which are only models of real-life situations, and *big worlds*, which we live in. The rightness and usefulness of Savage's theory depends largely on the question whether it is possible to transfer the construction of *a small world* to *a big world*. Is *a small world* an adequate representation of *a big world* and does it fulfill the axioms?

[8] The completeness means that any two objects are either exactly ordered or equivalent, while transitiveness guarantees that there are no cycles of exact preference. The S of completeness and transitiveness are included in the axiom P1.

Expected utility

The order of preference can be established on the basis of the expected value U (so called utility function) for the decisions. It means that:

$$f \succcurlyeq g \text{ if and only if } E(U(f)) \geq E(U(g)).$$

Two utility functions establishing the same order of preference are referred to as strategically equivalent. Otherwise, they are called strategically non-equvalent. The power of this result is rooted in the fact that it allows to attribute to preferences the function of real values defined on the results of these preferences. It could be used to help solve decision problems by identifying the utility function.

Example

Mr X is a student of marketing and management and his hobby is watching *Bay Watch*. The time has come to decide what path to take. Mr X has two alternatives: either a life-guard or a manager. Considering the options, Mr X takes two factors into account: Z which is health and B which is wealth (their lack will be marked NZ and NB, respectively). He thinks that being a life-guard is healthy but does not make you rich. On the other hand, a manager can be rich but his hard work and lifestyle are likely to lead

[9] Cf.e.g. Anscombe and Aumann (1963) and Fishburn (1967).

to his health deterioration. Let's assume that in our example there is a little chance of a life-guard becoming rich (0.1%). Thus, his career prospects are as follows: ZNB – 0.999, ZB – 0.001. On the other hand, however, we can assume that there is a little chance of a manager going bankrupt (as a result of his wrong decisions). Career prospects of a manager are thus as follows: NZ, B – 0.99 and NZ, NB – 0.01.

Supposing that the utility function U for Mr X is: $U(ZB) = 1$; $U(Z, NB) = 0.6$; $U(N, ZB) = 0.7$; $U(NZ, NB) = 0$, Mr X should choose to become a manager, since the expected utility of this decision is 0.693 and is higher than the expected utility of a life-guard's career, which stands at 0.6004.

Violation of axioms of Savage's theory

Allais'Paradox

Savage's model, and his Sure Thing Principle P2 in particular, was immediately attacked by a French economist Maurice Alais. He presented an example of paradoxical decision problem in which a decision-maker chooses a decision that violates the Sure Thing Principle.

Option	A ($p = 0.1$)	B ($p = 0.89$)	C ($p = 0.01$)
M	\$5,000,000	\$1,000,000	\$0
N	\$1,000,000	\$1,000,000	\$1,000,000
M'	\$5,000,000	\$0	\$0
N'	\$1,000,000	\$0	\$1,000,000

Most people choose M not N but also N' not M'. This violates the Sure Thing Principle as the same results (option B) should not influence the choice between the alternatives.

Ellsberg's Paradox

Although Ssavage's theory has two subjective functions (probability and utility) and seems difficult to test, in 1961 Ellsberg presented an example of a decision problem which contradicts the theory of subjectively expected utility. Ellsberg's paradox can be illustrated by the following:

Supposing we have an urn with 90 bowls: 30 red ones and 60 blue or green ones in an unknown proportion, we will consider the following game:

Option	Payoffs for drawing a ball of each color		
	Red	*Blue*	*Green*
F	\$100	\$0	\$0
G	\$0	\$100	\$0
F'	\$100	\$0	\$100
G'	\$0	\$100	\$100

Most of the pollees choose F not G and G' not F'. People who prefer F to G should also prefer F' to G', as the only difference is the result for the green bowl, which does not differenciate between F and G or F' to G'. If an individual prefers F to G, the theory claims that $U(\mathit{red}) > U(\mathit{blue})$, whereas if an individual prefers G' to F', $U(\mathit{red}) < U(\mathit{blue})$. Considering the preferences of the pollees, we arrive at a contradiction. Many people confirmed this paradoxical choice even if they knew Ellsberg's paradox. One of the possible explanations of this phenomenon is aversion to uncertainty. The other one is that decision-makers do not believe that urns in both cases are the same. This paradox falsifies the hypothesis that convictions can be represented by subjective probability.

Asymmetrical Domination

According to the axioms of Savage's theory, if acts f and g are given, the additional act h should not influence the preference between acts f and g. However, this proves untrue.

Let's consider the sales of beer. Brand X costs 1.80 zl per bottle and its quality is rated at 50. Brand Y costs 2.60 zl with the quality rated at 70. Some prefer X, some Y. Let's add brand Z costing 2.00 zl with the quality rated at 50. It is obvious that Z is worse than X. We could say that Z is dominated by X as it is cheaper and of the same quality. Thus, Z should not change the preferences between X and Y. However, it does. People tend to choose X more often when given the choice of X, Y and Z than when given the choice of X and Y only. Why? There seems to be another reason for such a choice. It is not clear whether X or Y is better but it is clear that X is better than Z. Thus, it can be said that X is at least better than some other brand, while Y is not.

The Reversal of Preference

Let us consider two holiday destinations:

A: average weather, average beach, average hotel, average water temperature, average nightlife.

B: plenty of sunshine, great beaches and reefs, luxurious hotel, freezing water, extremely strong winds, no nightlife.

33% of the pollees chose A, while 67% preferred B. Those who had booked two destinations, paid advance payments and had to choose between A and B made the following decisions: 52% abandoned A and 48% abandoned B. According to the theory of expected utility the preference to choose or abandon should be the same. There is the following explanation of this phenomenon: as much can be said in favour of B as against it.

The reversal of preference is explained in the following way: the choice is relative and depends on the way the question is asked. When pollees are asked what they want or what they do not want their attention is drawn to either positive or negative aspects. This mostly accounts for differences in preferences.

Conclusions

Savage's theory, as its author claims, could be viewed as an immature and superficial empirical theory of foreseeing human behaviour while making decisions. It can be applied only to limited fields and everybody can use it to predict some aspects of human behaviour. Concurrently, human behaviour often contradicts theories, sometimes to an outstanding degree. Usually, such results are attributed to chance or subconscious motives.

If we compare Savage's theory of decision and von Nauman and Morgenstern's theory of games, von Neuman and Morgenstern's theory of expected utility seems to be more applicable to games in which matrix of payments and players' choices from possible random strategies are known because of the construction of the game. Savage's theory of subjective utility seems more applicable for modelling games that are created by nature. In these games the agent has to shape his subjective convictions regarding both: payments (consequences) and strategic intentions of his opponents. Formally, Savage's model refers to a decision problem in which a single agent is engaged in a battle against impersonal forces of nature.

Both theories seem to be incomplete when applied to economic decision problems. This is because:

- expected (or subjectively expected) results of utility can contradict the market rating,
- neither of the theories takes temporal component,
- neither of the theories takes prior changes in consumption into account.

Savage's thought was the basis for other axiomatizations of theories of subjective probability and theories of utility, namely the works of Anscombe and Aumann (1963), Pratt, Raiff and Schlaifer (1964) and Fishburn (1967) among the others.

One of the more promising solutions seems to be the idea to represent individual convictions by non-additive probability.

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STUDIES IN LOGIC, GRAMMAR AND RHETORIC 5 (18) 2002

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REMARKS OF THE PSYCHOLOGIST ON HAYEK'S IDEAS ABOUT THE ROLE OF KNOWLEDGE IN ECONOMIC PLANNING

1. It is well known that Hayek, opted for decentralized, i.e. divided between many entities, economic planning, was strongly against central planning exercised by a single entity governing the whole economic system. An important argument for decentralized planning was the dispersion of information in society. As Hayek claims, it is the technical knowledge concerning a specific situation and a specific moment in time that plays a decisive role in effective economic planning. Noteworthy, this kind of knowledge is inaccessible for economic experts, but is accessible for individuals engaged in an economic process.

As Friedrich Hayek (1945) wrote in "The Use of Knowledge in Society", *American Economic Review*, *35*, 519-30: *If we can agree that the economic problem of society is mainly one of rapid adaptation to changes in the particular circumstances of time and place, it would seem to follow that the ultimate decisions must be left to the people who are familiar with these circumstances, who know directly of the relevant changes and of the resources immediately available to meet them.*

A psychological research on entrepreneurship gives Hayek's line of argument an empirical support: a considerable proportion of successful businessmen are people with very low formal education, but who, at the same time, are able to react quickly to specific information concerning time and place.

2. Incomplete and imperfect knowledge of people planning economic actions does not prevent making accurate decisions as long as there exists a simple indicator which enables people to coordinate their activities. And this is the PRICE that plays such a role in economy. Hayek gives an example of a new application of tin which suddenly makes it more sought-after. Tin consumers (and not exactly all of them) only need to know that some of

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tin they used to consume is now utilized in a new and more profitable way, and, consequently, they should use it in a more efficient way. This will induce appropriate actions: making investments in tin, searching for its substitutes, supplying of tin goods etc. All of it will have impact on the price of tin. What is important is that nobody needs to reconstruct the whole chain of events that influences the price. The price itself is the simplest and most efficient mechanism of spreading the economic information. Indeed, to take an accurate economic action noone has to trace the whole process that leads to a particular price.

The thesis about the importance of the simplest mechanisms of spreading information finds its empirical support in the psychological research. Starting from Herbert Simon, psychologists put emphasis on the fact that people have a limited capacity to process information. Consequently, human beings have to adopt the simplified rules of decision making, which Tversky and Kahneman referred to as heuristics. It is also known that an excess of information can deteriorate (instead of improving) the quality of human decisions. Ideas of Hayek fit very well to such a vision of human mind.

3. Yet, there is yet another intriguing problem, not mentioned by Hayek: not only the lack of knowledge, but even some cognitive illusions of economic agents may lead to the economic growth.

For example, the tendency to adopt illusions in the perception of risk and probabilities paradoxically quite frequently strengthens the willingness of entrepreneurs to get involved in risky ventures. Although the willingness to take up risky ventures may lead individual entrepreneurs to fail in business, undoubtedly it is the engine of the economic development of societies.

Another instructive example gives the way the investors act at stock exchange. On the level of individual decision making we can find an enormous number of illusions and biases. For instance, against the law of regression to average, the investors believe that the future will be the same as the past, and they are prone to buy the shares whose prices have recently risen, and avoid buying shares that recently have fallen down. Some of the researchers of the so-called behavioral finance try to demonstrate that these illusions are in fact the power that makes the stock market work. Moreover, they claim that due to these illusions stock markets function in such a way that all the new information about a company is immediately reflected in the price of its shares (i.e. market is *effective*).

STUDIES IN LOGIC, GRAMMAR AND RHETORIC 5 (18) 2002

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MATHEMATICAL METHODS ON COMMODITY EXCHANGES

1. Introduction

Using of various often quite sophisticated mathematical methods in different branches of economy is now so common that mathematics became an indispensable element of any economical curriculum. In the following we concentrate our attention on some mathematical models and methods that can be applied to the rational option pricing on commodity exchanges. Commodity markets are quite specific and full of different derivative instruments. In Poland there are two main "players" in the field, namely: Giełda Poznańska (GP) and Warszawska Giełda Towarowa (WGT). In general, on the commodity exchanges we can distinguish the following basic contracts (instruments):

- Options: giving rights to buy or sell a commodity,
- Forward: being an obligation for buying and taking/delivering an asset,
- Futures: transactions providing security against unfavorable changes of prices and not a real realization of a contract.

An *option* is a contract that gives the right to buy or sell a prescribed asset for a given price at a prescribed (expiration) time (European option) or at any time prior the expiration date (American option). Two main types of options are *call* and *put* options.

- Call option – the holder of the call option has the right but not the obligation to buy a given asset for a given price. The writer of the call option must sell the asset for a fixed exercise (or strike) price if the holder decides to buy it.
- Put option – the holder of the put option has the right but not the obligation to sell an asset for a given price. The writer of the put option must buy the asset under the prescribed conditions if the holder decides to sell it.

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The question of how much should be paid for the mentioned right or, in other words, what is a reasonable value of an option is the main issue discussed in this paper.

2. Statistics of commodity exchanges in Poland

As already mentioned, in Poland there are two institutions (GP and WGT) dealing with various derivative contracts and instruments for commodities. The first transaction of this sort was performed in 1995 at GP – 100 options for porkhalf were issued. The second emission took place in February 1996 – again 200 contracts for porkhalf. In both cases the writer was Agencja Rynku Rolnego (ARR). Almost all (198) options were actually sold. In 1997 options for milling wheat were emitted by a private business. Since 1998 transactions dealing with contracts on milling and feed wheat have been performed. In 1999 contracts on live hog were added. On WGT first contracts were futures contracts on currency exchange rates for USD/PLN and DEM/PLN as well as futures for milling and feed wheat. In March 1999 futures contracts for interest rates (one and three months) WIBOR have been introduced. In May 1999 the contract DEM/PLN was replaced with EUR/PLN. Also a new commodity futures contract for live hog has been introduced. All these futures contracts were emitted by ARR.

Table 1. Statistics of transactions on GP

Year	Number of clearing houses	Volume in thousands PLN	Number of transactions	
			cash	derivatives
1991	1			
1992	16	6104,1	246	
1993	22	65318,0	1328	
1994	20	169190,7	1090	
1995	33	269393,4	3118	111
1996	34	426298,0	3842	376
1997	27	362675,1	2914	204
1998	19	231341,6	4380	1119
1999	20	551874,9	12174	1607
2000	20	964932,4	15750	118
2001 (till 30.08)	21	277034,9	3558	–

Source: Prepared by the authors

The number of transactions was not large – after the dynamic growth in the period of 1998-1999 it drastically decreased in 2000.

On WGT trading of commodity options began on June 12, 1997. Options available on WGT are standardized:

- Fixed quantity of the basic asset (20 tons of wheat, 50 tons of corn, 5 tons porkhalf or beef carcasses).
- Fixed duration of 4, 7, 8, 13 or 26 weeks, starting from the emission date.
- Registered and settled via the WGT clearing house.
- There is a commission – the fee equal to the option premium.

In 1997 on WGT the total volume of derivatives traded was given by a six-digit figure: 113154 PLN. In 1998, 1400 options were emitted, with the total value of 362963 PLN. Many of them were not purchased at all. In 1999 the American call options for milling wheat were issued. The total volume was worth much less – 11994 PLN.

3. Forecasting of commodity prices

There are many methods that could be used to forecast prices for agricultural goods, being the underlying assets for any derivative contracts we discuss in this paper. Let us just mention the most common examples:

- Single-equation econometric models of different forms and types, including:
  - linear: $y_i = \beta_0 + \beta_1 x_{1i} + \ldots + \beta_k x_{ki} + \varepsilon_i$
  - nonlinear with respect to explanatory variables, e.g. $y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \varepsilon_i$
  - nonlinear with respect to both explanatory and parameters, but a transformation to linear models is possible, e.g. $y_i = \beta_0 \cdot x_{1i}^{\beta_1} \cdot x_{2i}^{\beta_2} + \varepsilon_i$
  - fully nonlinear, e.g. $y_i = \beta_1 + \beta_2 e^{\beta_3 \cdot x_{1i}} + \varepsilon_i$
- multi-equations econometric models:

$$Y_1 = \sum_{i=2}^{m} \beta_{1i} Y_i + \sum_{j=1}^{k} \lambda_{1j} Z_j + \varepsilon_1$$

$$Y_2 = \sum_{i=1, i\neq 2}^{m} \beta_{2i} Y_i + \sum_{j=1}^{k} \lambda_{2j} Z_j + \varepsilon_2$$

....

$$Y_m = \sum_{i=1}^{m-1} \beta_{mi} Y_i + \sum_{j=1}^{k} \lambda_{mj} Z_j + \varepsilon_m$$

- various types of neural networks
- nonlinear prediction models based on the deterministic chaos theory

In this paper we do not discuss further the problem of forecasting of the basic agricultural assets. We just assume that one or the other method can be used to model the price dynamics of the underlying commodity. Nevertheless, it is an important and nontrivial problem. In the following we introduce two simple versions of stochastic dynamics of underlying assets when presenting two fundamental and quite popular models frequently used for option pricing.

4. Basic models of option pricing

Here the Black-Scholes model and a binomial model of Cox-Ross--Rubinstein are briefly introduced. They will be elaborated in more details in the forthcoming sections. Later on they will also be used to analyze and illustrate the option valuation with data coming from Polish commodity markets.

The Black-Scholes model assumes (among other things) that prices of underlying assets are the subject to the continuous changes. The model is mainly used to European call and put options for the stock shares, stock indices, and the currency exchange rates but it can also be applied for American options pricing if underlying assets pay no dividends.

The Cox-Ross-Rubinstein model assumes (somewhat more realistically) that prices of the assets can change in a discrete way. In other words, a continuous stochastic process is replaced by a discrete random walk. The model is often used for pricing European options for stock shares, instruments with fixed dividend, currency exchange rates, indices, and for American options.

It should be mentioned that these two basic models are just the simplest examples of a much broader spectrum of methods and approaches used for option pricing. They are, however, very important examples, not only from the theoretical perspective but also from the practical point of view. Many other methods are just straightforward generalizations of them.

5. Black-Scholes model

Now classical, Black-Scholes model is based on the following assumptions:

- assets prices undergo a stochastic process of Ito's type (the Brownian motion),

- there are no overheads, taxes, and other transaction costs,
- assets pay no dividends during the time of option validity,
- there are no risk-free arbitrage opportunities possible,
- the so-called short selling is allowed,
- buying and selling of assets are possible continuously, i.e., assets can be traded in a continuous way
- all market participants can rent and invest money with the same risk--free interest rate,
- in the short term, a risk-free interest rate is constant.

Obviously, the option value V at time t is given by:

$$V(s,t) = \max(S(t) - W, 0)$$

where $S(t)$ denotes a market price of a given asset at time t and W is the exercise price.

A fundamental problem is to find a formula describing the option value (price) V as a function of an asset price $S(t)$ at an arbitrary given time moment t. Any solution to this problem requires a market model, which describes dynamics of prices of a given asset. Within the Black-Scholes approach we assume the following model of the asset (stochastic) dynamics:

$$dS(t) = \sigma S(t)dX(t) + \mu S(t)dt$$

where $dX(t)$ is a Wiener stochastic. Parameters μ and σ characterize the market.

Due to the presence of the stochastic element dX, the asset price S is a random variable at any given time moment. Still it is possible, and this is a great achievement of Black and Scholes, to construct a deterministic equation relating the option value to time and the asset price.

The key point is a construction of a secure portfolio consisting of a number of assets and a call option written for it. At any time t the value of such a portfolio is given by:

$$\Pi(t) = V(S,t) - nS(t)$$

This does not depend on the current price of the asset only if:

$$n = \frac{\partial V}{\partial S}$$

Such a situation corresponds to the so-called perfect hedging. Now we can make use of the assumption about the lack of arbitrage. Any potential gain of the portfolio value should be equal to the gain obtained from the risk-free investment:

$$r\Pi dt$$

From this we immediately obtain an equation describing dynamics of the option value:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

From the mathematical point of view it is a partial differential equation of the parabolic type. There are two parameters: a risk-free interest rate r and volatility of the asset price σ. The solution to this equation (corresponding to the proper initial and boundary conditions) directly leads to the Black-Scholes formula for the option value V:

$$V(S,t) = SN(d) - We^{-r(T-t)}N(d - \sigma\sqrt{T-t})$$

Here N is given by the well-known integral:

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}y^2} dy$$

Parameter d is defined as:

$$d = \frac{\ln\left(\frac{S}{W}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

Thus for a given time t the option value depends on two variables (time left to the expiration date $T - t$ and the current price of the asset S) and three parameters (risk-free interest rate r, volatility of the asset price σ and the exercise price W). One of the most important problems in the process of option pricing is estimation of σ.

6. Cox-Ross-Rubinstein binomial model

Some of the basic assumptions underlying the binomial model of Cox--Ross-Rubinstein for option pricing are the same as for the previously discussed Black-Scholes model. These are:

- there are no overheads, taxes, and other transaction costs,
- assets pay no dividends during the time of option validity,
- there are no risk-free arbitrage opportunities possible,

- all market participants can rent and invest money with the same risk--free interest rate,
- in the short term, a risk-free interest rate is constant.

There are, of course, also big differences. First of all, in the binomial model assets prices undergo a discrete random walk, what means that buying and selling of assets are possible only at some fixed time moments. Moreover, we assume that at any of these particular times, the asset price can either do up or down by a fixed amount and with a given constant probability.

Fortunately, the binomial model is very flexible and can easily accommodate a constant dividend paid by the underlying assets.

If there are no dividends, after the first step Δt we have just two possibilities: the asset price goes up to $S \cdot u$ or down to $S \cdot d$. After the second step at time $2 \cdot \Delta t$ there are already three possibilities: $S \cdot u^2$, S and $S \cdot d^2$. Generalizing this to "i" steps we see that after $i \cdot \Delta t$ there are $i + 1$ possible situations, which can be reached using 2^i ways. Any possible (reachable) price level can be computed using the following expression:

$$S \cdot u^j \cdot d^{i-j} \quad \text{for } j = 0, 1, 2, \ldots, i$$

Under the above assumptions the quantities u, d, p are given by:

$$u = e^{\sigma \cdot \sqrt{\Delta t}}$$

$$d = \frac{1}{u}$$

$$p = \frac{e^{r \cdot \Delta t} - d}{u - d}$$

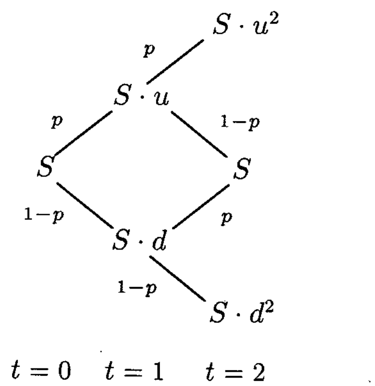
where:

- σ – volatility of an asset price (measured by the standard deviation – usually computed as the historical volatility),
- r – risk-free interest rate,
- Δt – length of the time interval.

The parameters p, u and d have to take into account proper values of the average and variance for an asset price for the time period Δt (see Fig. 1).

It is worth noting that the Cox-Ross-Rubinstein model leads to a very effective computer code and usually provides a good quality numerical answers.

Figure 1. An idea leading to the binomial model



Source: Prepared by the authors

7. Case studies – empirical examples

Here we present a strategy "in action" of using these methods for the purpose of the derivatives pricing on Polish commodity markets. Our studies are based on empirical data obtained from both GP and WGT.

First, using empirical data from GP, we estimate the values of a futures contract and a call option. Then we present the analysis of European option pricing using data from WGT.

Example 1.

First, consider a futures contract, valid for three months, issued for 20 tons of rye, emitted in June 2000, with the following parameters (to perform the actual computations we used the software *The Black-Scholes And Beyond Interactive Toolkit*):

S – 426,50 PLN/ton (a price of the underlying asset, an average price in June 2000),
W – 426,50 PLN/ton (an exercise price),
r – 17% (a risk-free interest rate),
T – 90 days (time to the expiration date),
σ – 25,7% (historical volatility estimated using the data from the last 12 time periods)

To estimate the option premium both Black-Scholes and Cox-Ross-Rubinstein models were used. The results are summarized in Table 2.

According to the Black-Scholes model, the option value was computed to be 31,14 PLN. A very similar number was obtained using the binomial

Table 2. Results of call option pricing for rye

Method	*Black-Scholes model*	Binomial model (n = 200 steps)	
Type of option	European option	European option	American option
Option value (PLN/ton)	31,14	31,11	31,11
Delta	0,65	0,65	0,65
Gamma	0,01	0,01	0,01
Theta	−0,23	−0,23	−0,23
Vega	0,78	0,78	0,78
Rho	60,94	60,93	60,93

Source: Prepared by the authors

model – 31,11 zł. In September 2000 (the expiration time), the average price for rye on the cash market was equal to 412,50 PLN/ton. Of course, it was unreasonable to exercise the call option at that time because it would generate the loss of 280 PLN per option.

The five "Greek" parameters listed in the table 2, provide in a compact and easily accessible form some additional information and measure the degree of dependence of the option price on the asset price, the time left to the expiration date, volatility, etc.

Example 2.

In June 5, 1997, ARR issued on GP a call option for 10 tons of porkhalfs, the expiration date being August 5, 1997. The initial option price proposed by ARR was 300 PLN. The actually negotiated average price was higher – 1210 PLN.

Input data:
S – 4,90 PLN/kg (a price of the underlying asset),
W – 5,20 PLN/kg (an exercise price),
r – 19% (a risk-free interest rate),
T – 60 days (time to the expiration date),
σ – 22% (historical volatility),

The Black-Scholes model produced the following value for the option premium:
V – 0,12 PLN/kg (1200 PLN for each option),
σ_{imp} – 22,59% (implied volatility for V = 1210 PLN),

Historical volatility was computed using the data from GP, giving σ = 22%. As the result of using the Black-Scholes formula we get the option premium of 0,12 PLN/kg, what gives together 1200 PLN per option issued for 10 tons. This is more than three times more that the initial (proposed) price and by just 10 PLN less than the average negotiated price. Then we estimated the implied volatility obtaining (for average premium 1210 PLN) 22,59%. This number is quite close to the historical volatility.

Example 3.

In June 23, 1998, on WGT, European call options for milling wheat valid for 8 weeks were emitted.

Input data:

S – 490 PLN/ton (a price of the underlying asset),
W – 500 PLN/ton (an exercise price),
r – 17% (a risk-free interest rate),
T – 56 days (time to the expiration date),
σ – 18,5% (historical volatility),

Using the Black-Scholes model we obtained the following results:

V – 16,7 PLN/kg (an actual price 12,5 PLN/kg, loss of 4,2 PLN/kg),
σ_{imp} – 13,9%.

In this case the actually negotiated exercise price was much less that the option value suggested by the model.

8. Brief Summary

The development of Polish futures and options markets, where various derivative products could be traded, is rather slow. There are several reasons for this unsatisfactory situation. First of all, the ARR plays an absolutely dominant role in the intervention market. This should change after the access of Poland to the European Community, allowing the derivatives trading to grow up dynamically. We believe that the mathematical methods of option pricing, presented in this paper, will still be useful in determining the option value. Our investigations show that despite the fact that some of the assumptions underlying both models are apparently not satisfying, both methods can be fruitfully used in practical estimations at the Polish commodity markets. Of course, it is possible to generalize the models, to relax some of the constraints, and to use a bit more realistic assumptions. Still both Black-Scholes and Cox-Ross-Rubinstein models will remain popular as the simplest examples of very useful and fruitful theoretical simplifications, providing a good starting point for further investigations.

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STUDIES IN LOGIC, GRAMMAR AND RHETORIC 5 (18) 2002

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THE DYNAMICS OF NONLINEAR SYSTEMS

Introduction

Nonlinear dynamics, commonly called the chaos theory, changes the scientific way of looking at the dynamics of natural and social systems. Since these changes are deep and great in number, it is impossible to discuss them all in one paper. In this introduction I will therefore try to show how the approach of physicists to dynamical systems has changed. It has led to the change of the role of physics amongst the other natural sciences. I shall start with the popular view of physics.

The success of physics as the most precise, fundamental natural science, held up as a model for other sciences (and not only the natural ones) was based on the fact that by using developed mathematical theories and making precise measurements, physics was able to describe, understand and explain the properties and behavior of many important kinds of bodies and dynamical systems, such as the Solar System, simple mechanical systems (e.g. the clock), atoms and so on. The results of physics considerably surpassed the achievements of scientists in other sciences, particularly in biology, geology, psychology, etc. Because of this, fundamental physical theories, such as classical mechanics and electrodynamics, were the model of scientific knowledge and other domains of science tried to reach a comparable level of generality and precision.

When studying physics I was proud to be learning such a perfect science, the ideal of scientific method, and I was not conscious that this picture of physics was not consistent with the actual practice. That inconsistency follows from the fact that even in general physical theories, which are mathematically well worked-out, there are not many phenomena which can be described and explained in a precise, theoretical way. Let us look from this viewpoint at mechanics for instance. Newton's equations give the universal

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dependence of motion upon forces, but in the monographs on mechanics we find only a few simple situations in which the equations really describe what is going on and which can be solved exactly. Without precise solutions nobody can predict the future behavior of the process. So we have two main limitations of physical theories: the phenomena investigated may be too complicated to be fully modeled and even if we have a model, we may not be able to solve its equations. When one learns physics, we are hardly ever aware of these limitations because the lectures are devoted to those problems which have been satisfactory and generally solved. Thus during classes one learns how to solve examples that can be solved. In virtue of this, I was sure, after I finished studying physics, that it was the powerful general science. Through investigating philosophical and methodological problems of physics I was slowly led to the conclusion that physics was actually not so powerful and that it had important limitations. Those are not limitations connected with inaccuracy of observations, often discussed by physicists, but with its theoretical methods and equations. We will examine them in connection with the theory of the Solar System.

The first well known difficult problem is that the equations of motion for bodies interacting under a gravitational force can be solved only for two bodies. Such bodies move on ellipses around a common center of mass. It is easy to write the equations of motion for systems with many bodies: these equations have, however, no analytical solutions. The so-called reduced Hill's problem is the simplest system that has no precise solution. The system consists of two big interacting bodies moving around their center of mass with the third small body moving in their common gravitational field. The third body is so small that its action on the larger bodies can be neglected. It moves in the well-known regular field of its two neighbors. Its motion is not always regular because there are areas in which the forces generated by the larger bodies balance one another and a small change of position results in the enormous change of the motion and the trajectory of the smaller body. This instability of the motion was discovered by H. Poincaré in 1892, who called this kind of motion the homoclinic tangle (Stewart, ch. 4). The example shows how a small complication of the system, the addition of a small body to two larger bodies moving in a regular way, leads to an essential change of the motion. The motion becomes complicated and unpredictable. Having discovered this, Poincaré was not able to pursue the study of it as at that time the reduced Hill's problem was too difficult to be described and analyzed precisely. Now, thanks to the computer, it is well understood. One can imagine how complicated the motion of ten bodies interacting under gravity would be. The dynamics of the Solar System is

simple because the mass of the Sun is so much bigger than masses of planets and the planets are far one from another. Because of this, one can separately study the motion of each planet with the Sun.

In those areas of physics investigating more complicated systems, such as hydrodynamics and atomic physics, computational limitations of even simple models are well-known and it narrows the area of their effective applications. The theory is in principle general but the range of its efficient models is much smaller. Thus we see that the important and fascinating successes of physics, in fact, comprise a narrow domain of natural phenomena. Most observed phenomena are too complicated for the application of simple physical models to be able to yield precise results. One has to use simplified models, approximate methods and inaccurate descriptions. The complexity of natural processes was the obstacle in the development of science. Of course, scientists could not give up investigating complex phenomena because of their importance for us. Without understanding them people would not be able to act and to develop technology. The study of complex systems conducted over the centuries has produced many important results and about forty years ago they began to come together to create a new universal domain of research called the theory of chaos. The theory offers effective, precise methods of complex systems analysis. It is a mixture of mathematical, empirical and computational methods and results. In order to describe its achievements and possibilities I shall concentrate my attention on two problems: the instability and complexity of motion of simple systems and the order appearing in the behavior of complex systems.

The role of instability in the behavior of dynamic systems

I start with a statement that seemed obvious not long ago: a simple material system should act in a simple way. Led by this principle, scientists tried to study the simplest physical, chemical and biological structures, because their behavior should be equally simple and intelligible. However, it can be easily shown that a simple system, for instance, a mechanical one, does not need to act in a simple way. A good example is a big pendulum on the end of which a small pendulum is hung. Each of them separately works in a simple, predictable way but their combination is an irregular unpredictable system. The small pendulum disturbs the motion of the big pendulum, but itself also behaves in a complex way because its hanging point is moving. Their motion is given by two interconnected differential equations which can be solved only approximately, yielding irregular, complicated solutions. The simplicity of the structure of the system does not imply the simplicity of its dynamics.

If we wish to understand the features of the dynamics of complex systems, we should look at them in a new way. Classical physics, first of all, tried to solve the equations of motion in order to describe quantitatively the motion of the system studied: a planet, a pendulum, colliding balls, etc. The motion of a complex system cannot be known precisely so we pose questions concerning its kind and properties. The most important question concerns the stability of the motion. Stability means that small perturbations of the motion result in small, slowly increasing changes of the trajectory. The systems which are the most important for us, such as the motion of Earth around the Sun or the motion of a car, are stable but many important processes, such as atmospheric phenomena, are unstable. Another problem concerning the dynamics of complex systems is the kind of motion realized by the system. That motion can neither be described nor predicted, as it is too complicated. It can, however, be characterized approximately and quantitatively. For example, by studying the behavior of a system with friction one can easily predict that after some time it will stop if there is no energy inflow from outside.

The chaos theory uses specific concepts to examine such problems. The most important is the concept of phase space (Tempczyk, pp. 34-37). It is the space of parameters completely describing the motion of a given system. In classical mechanics, in studying the motion of a body we usually use the coordinates of its position, but this does not provide a full description since bodies can move on one trajectory with different velocities. Therefore, the phase space of material point is built from positions and velocities. For formal reasons physicists use momenta instead of velocities. Momentum is the product of mass and velocity of the body. The advantage of phase space is that it contains the entire history of the motion of the system – its trajectory. Because of the uniqueness of the solutions of the equations of motion, trajectories cannot cross. They are lines resembling the lines of the flow of water in a river. Looking at trajectory families, which are classes of the equations of motion solutions, one can answer questions concerning the kind of motion. In the case of stable motion, neighboring trajectories disperse slowly and are rather regular. If the motion is unstable, trajectories close at the beginning separate rapidly, frequently changing their direction in the phase space. If the motion of a typical system ends in the same way, for instance, by becoming slower and slower, then all trajectories tend to the same point or area, this being called an attractor.

An often used tool of the chaos theory is the iteration procedure. We take any starting point and study how it changes after 1 second, 2 seconds, 3 seconds and so on. By observing those points in the phase space we see how the system moves: whether its motion is regular, cyclic or chaotic. Observations of this kind yield a lot of information about the dynamics though we do not possess analytical solutions of the equations of motion. Such approach to the dynamics is called a qualitative theory of differential equations.

The next key concept of the theory of chaos is linearity. A system is linear if the differential or algebraic equations describing it are linear. Systems of linear equations are easy to solve, which is why the theory of those equations was well developed in the 19th century. It was used by the empirical sciences and through it linear processes became well understood. A process is linear when its parts act in the same way, independently of their surroundings and other parts. An electric field is linear. Each charge generates a defined field and the global field is the vector sum of all partial fields. One might say that the electric charge 'does not know' its environment – the system which it belongs to – and its field is always the same. The classical Newtonian theory of gravitation is likewise linear. Linear systems are easily decomposed into parts. Scientists study those parts and then reconstruct the whole. This approach is ineffective with respect to nonlinear systems as their parts adapt to the environment and their behavior is unpredictable if they are examined in separation. Most natural technical and social processes are nonlinear and non-linearity presented a substantial obstacle for science even forty years ago (Tempczyk, pp. 24-26).

Let us return to the behavior of simple systems. Their dynamics need not be simple. The Lorenz gas is an example. It is a model of electron motion in a crystal. Electrons move along straight lines and collide with atoms that are like balls arranged in a regular way. The collisions electrons make with atoms are unstable because an electron moving towards the center of the atom can turn right or left, depending on small deviations of its trajectory: Two electrons initially moving along close paths fly in different directions after colliding and their future is different. Because of this, the movements in the Lorenz gas are unstable. It is a linear system and its instability has a geometrical origin.

A well known example of simple system with complex dynamics is the system described by the logistic equation, first studied by R. May and next by M. Feigenbaum. Feigenbaum investigated the behavior of trajectories given by the simple square equation:

$$x_{n+1} = kx_n(1 - x_n) = f(x_n)$$

depending on the parameter k. It is the function that maps the interval [0,1] into itself, if $0 < k < 4$. Feigenbaum wanted to work out the motion beginning from any point x_0.

For $0 < k < 1$ the answer is simple as always $x_{n+1} < x_n$ and after many steps the value of x_n is close to 0. So 0 is the unique attractor of the system. We can imagine that x describes the population of grasshoppers in a meadow, where 1 corresponds with the maximum number of the insects in the meadow and k is the coefficient of their reproductiveness. With $k < 1$ grasshoppers reproduce too little to survive as each generation is smaller then the preceding one. They, therefore, perish.

For $1 < k < 3$ the situation is also simple. There exists the stable point $x_k = 1 - 1/k$ which is the attractor as all trajectories, except the one starting from $x = 0$, tend to it. For R. May, the biologist who used the logistic equation to describe population dynamics, the result was obvious. It proved that each population will tend to an state of equilibrium, depending on the reproductiveness of animals and environmental conditions. It was consistent with the scientific view on the nature of biological equilibrium.

The behavior of the systems changes radically when $k > 3$. The point $x_k = 1 - 1/k$ is still stable, but the value of $|dx/dt|$ becomes greater than 1 in its neighborhood causing x_k to change from an attractor to a repeller [repulsion point]. Instead of it appear two adjoint points x_1, x_2, such that $x_1 = f(x_2)$ and $x_2 = f(x_1)$ and these take on the role of the attractor. Each trajectory approaches one of them and oscillates with them in a two-element cycle. The attractor point changes into a two-element attracting cycle. The pair x_1, x_2 attract neighboring trajectories because the composition of functions $f(f(x_1))$ and $f(f(x_2))$ has the absolute value of its derivative smaller then 1. This situation changes again for $k = 1 + \sqrt{6}$. For this value each of the two branches bifurcate and there arises an attracting four-element cycle. Once again, all trajectories approach those points and jump with them in a definite order. It is easy to see that a further increase of k generates an 8-element cycle, a 16-element cycle and so on. At the limit value of $k = 3.5699456$ the cycle becomes infinite and one observes the characteristic picture of the Feigenbaum bifurcations (Tempczyk, p. 63).

Feigenbaum noticed, by studying the problem on his calculator, that successive bifurcation points become closer and closer and that the proportion of their distance remains constant. He calculated the constant $\delta = 4.6692016091$, which has been called the Feigenbaum constant in honour of him. Mathematicians were initially sure that the constant was related to the logistic function, but when Feigenbaum published his results, scientists from Los Alamos, N. Metropolis and M. and P. Steins, studied the dynamics of another function $x_{n+1} = rx_n \sin \pi x_n$, obtaining the same bifurcation scheme and, what was more important, the same constant. Many kinds of functions have since been examined and in each case the bifurcation structure has had the same constant δ. A new universal number appeared in mathematics.

At that time Feigenbaum's discovery was semi-empirical as there was no mathematical theory describing and explaining the behavior of attractors and their bifurcation points for a given function $f(x)$. Such a theory was elaborated over the course of several years and it was proved that the Feigenbaum bifurcation scheme was universal for functions having one distinctive maximum in the interval [0,1]. Those functions with another shape, for instance, those with two maxima, have another way of arising and branching attractors. The currently developed bifurcation theory is presented in various monographs, for example, in the book by Schuster (ch. 3). It is an excellent example of mathematical theory created for the description of complex systems. There are hidden interesting universal properties in their complicated and hard-to-predict behavior. This kind of mathematics is useful in the study of biological, economic, and mechanical systems. In economic contexts, it reminds me of the once-popular theory of cyclic crises in capitalism. Marxist economists claimed that capitalists invested too much, causing overproduction and cyclic crises leading to reduction of the number of firms. Then the next boom appears, capitalists invest too much and the situation repeats. It resembles the bifurcation scheme for $k > 3$, where a two-element attracting cycle shapes the dynamics.

Complex systems with simple action

The chaos theory shows two aspects of complexity. One of them was described above. Now we shall discuss the second one – the arranging action of non-linearity. A nonlinear system is one in which particular elements adapt to the environment and the whole. The consequence of such a global adaptation is that there arises a global order which is different from the order of local interactions and exceeds their diversity. Such global dynamic structures have been studied by the empirical sciences: for example, patterns of flowing water, tornadoes, living organisms, and ecological systems. These studies were difficult and imprecise because of the lack of theoretical tools and enormous complexity of those systems. From time to time there, however, appeared curious and important results that will be the subject of the discussion below.

Benard cells provide an example of complex structures that organize themselves as a consequence of the process of heat flowing into them. Benard was a French physicist who in 1900 discovered and accurately studied the global order emerging in shallow water heated from below. It is a process familiar from everyday experience. Initially, when the temperature of the heated bottom is relatively low, warm water rises up as one volume, loses its heat at the surface and sinks as it becomes cooler and heavier. When the temperature of the bottom increases, the process becomes quicker and more intensive (Tempczyk, pp. 81-82). At a certain moment, when the temperature reaches a critical value, there is a rapid change of the movement of the water and the transfer of heat. Short, parallel cylinders come into existence which rotate in such a way that the friction of their neighbors becomes as small as possible. Water moving up and down in the cylinders carries heat quicker and with lesser friction than previously and the heat flow is more effective and less chaotic. At first, the Benard cells are stable: small fluctuations and disturbances do not destroy them. However, the increase of the heating temperature causes an increase of their rotation speed and at a certain point the cells become unstable. They start to oscillate and in the end the structure disintegrates. The water motion becomes chaotic again. Benard studied the process, photographed the cells and published the results. During dozen of years physicists tried to formulate the theory of how they arise but were unsuccessful. In the monograph by Chandrasekhar (1961) devoted to hydrodynamics, the discussion of Benard cells and their theories occupies a big part of the book. They are a good example showing how the global order arranges and facilitates the course of the process: in this case, the flow of heat.

In 1963 an American meteorologist, E. Lorenz, used the model of Benard cells to describe the dynamics of processes taking place in the atmosphere over ground heated by the sun's rays. The systems resemble those of a liquid heated from below, so Lorenz elaborated a similar model and wrote three equations describing its dynamics (Schuster, ch. 1):

$$dX/dt = -\delta X + \delta Y$$
$$dY/dt = rX - Y - XZ$$
$$dZ/dt = XY - bZ.$$

The three parameters of the model are: X the velocity of the air circulation; Y – the difference of the temperatures of the air going up and down; Z is proportional to the deviation of the temperature from the equilibrium state. Lorenz had a computer at his disposal and worked out a program solving his equations. It helped him to discover two essential features of those equations.

The first property, called by Lorenz 'the butterfly effect', is the instability of the solutions. The computer calculated the values of Y twice. The starting value of Y was slightly simplified the second time and it resulted in a completely different shape of the Y function. Lorenz came to the conclusion that atmospheric phenomena are unstable and the weather forecasting cannot be done effectively for periods of time longer than a few days because the errors are too great compared to the parameters calculated.

More important was the second Lorenz's discovery – that of an attractor. Lorenz decided to study the long-term behavior of his system. He set the computer in motion and left it to work for a long period of time, having no idea what the results would be. After some time the solutions, the trajectories in the 3-dimensional space for given X, Y, Z parameters, started to arrange themselves in a 2-dimensional pattern of two leaves, now familiar from books on the chaos theory. The trajectory first wandered along on one of them, moving on circles, then rapidly jumped to the other one, again drawing circles, then jumped again back to the first leaf, and so on. The number of turns on one leaf was unpredictable. It was of a completely accidental nature, even though the system worked according to strict deterministic equations. The same attractor arose for different trajectories starting from different initial conditions and thus had a universal character. I am not going to describe the structure of the Lorenz attractor as it is well known (Schuster, ch. 5; Tempczyk, pp. 67-69), I would like to emphasize rather its great importance for science. In 1963 the idea of such an area attracting neighbor trajectories was incomprehensible and Lorenz's colleagues treated it simply as a by-product of the calculating procedure used. Lorenz published his results in a professional meteorological journal and stopped researching the problem. Ten years passed before mathematicians and naturalists began to understand the role of attractors. They then started to look for them in nature. Lorenz's work was rediscovered and its author became famous. Mathematicians found precise constructions leading to attractors, such as the Rossler and Henon attractors. Scientists started to discover attractors in data describing the dynamics of processes taking place in nature and society.

Presently, there is no mathematical theory of attractors. Mathematicians are not able to decide if given equations have attractors and for which values of the controlling parameter. One has to make one's calculations and observe whether the solutions reveal regularities corresponding to an attractor. One thing is certain. Trajectories can approach one another in the phase space to create an attractor only when there is inside the system the dissipation of energy supplied from outside. This explains why the energy-conserving Hamiltonian systems have no attractors. In the case of the Lorenz systems, energy is carried by the sun's rays and then transferred to higher levels of the atmosphere.

There are two methods for looking for the attractors of dynamic empirical systems. One of them consists in working with a mathematical model of the system. The equations of motion are solved regaedless whether they have an attractor or not. The history of the Lorenz equations was of this kind. Very often, however, scientists have no mathematical model of the phenomena under study but they do have a lot of empirical data which they try to order. Attractors are a type of order which is very difficult to observe. In 1981 F. Takens worked out a method of discovering attractors with delayed time series (Schuster, ch. 5.3). The method was successfully applied by a team of physicists led by R. Shaw to the study of a dripping faucet (Crunfield... 1986). They obtained interesting results which they published in Scientific American. They measured the temporal distance between succeeding drops and using Takens's method and they acquired a three-dimensional picture of the attractor. Next, the scientists elaborated the mathematical model of the process of the drop falling off and by solving its equations they found the same picture of the attractor. This example proved the effectiveness of the Takens's method. It is now widely used to search for regularities in biological, demographic, and physical systems.

Perspectives – the chaos theory in social sciences

In conclusion, I shall analyze the new possibilities the chaos theory gives to the social sciences. Its methods and results enable scientists to study in a new and effective way the behavior of complex systems which are too complicated to be analyzed by classical tools. The application of those methods has brought enormous progress in many well-developed domains, such as hydrodynamics, physics, chemistry and biology. Those are fields of science which study both simple and complex systems. However, the methods are most promising in those fields of research in which scientists are from the beginning dealing with complex phenomena and where they cannot use simplified models as applied in classical science. Such a situation is typical for sociology and economics with the result that in those sciences standard mathematical models based on differential equations are not efficient and their possibilities for gaining knowledge are fairly limited. Scientists have to use methods taking into account the high level of complexity of the processes under study. There are two different ways they can take.

The first one is the construction of nonlinear mathematical models of phenomena. The models help to understand and explain some of astonishing properties of self-organizing systems. They have been successfully applied by many researches investigating social processes. The application of such models is a straightforward affair; if there is a possibility of modeling mathematically complex processes, then such methods are used when necessary. More promising and of greater generality is the second way of analyzing complexity – the search for attractors in big databases lacking formal models. It is more general than mathematical modeling as one can draw important conclusions about complex processes while having no idea about the nature and course of those processes.

When we study complex processes whose representation requires many parameters, then any formal model of them is of necessity approximate. The use of such a model is efficient only when it enables one to grasp the essential properties of phenomena under study and to predict their future. Having the model, one can complicate it, making it closer and closer to the real process and obtaining better results. The agreement of theoretical predictions with observations proves that the scientist is heading in the right direction and that his theory adequately represents the reality. This route is, however, not open when there is no theory as all approximate models will give inaccurate results and researchers do not know how to describe theoretically the processes analyzed. They can then only use model-independent methods of data analysis. The most sophisticated method is the search for attractors in big sets of empirical data. The method helps to discover regularities hidden in the chaos of local relations and complex behavior. It is widely used by economists, sociologists and psychologists.

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